Format and topics
Exam 1, Math 131A

General information. Exam 1 will be a timed test of 75 minutes, covering Sections 3, 4, 5, 7, 8, and 9 of the text, as well as the handout on asymptotics. No books, notes, calculators, etc., are allowed. Most of the exam will rely on understanding the problem sets and the definitions and theorems that lie behind them. If you can do all of the homework, and you know and understand all of the definitions and the statements of all of the theorems we've studied, you should be in good shape.

You should not spend time memorizing proofs of theorems from the book, though understanding those proofs does help you understand the theorems. On the other hand, you should defintely spend time memorizing the *statements* of the important theorems in the text.

Types of questions. There are four types of questions that may appear on exams in this class, namely:

1. Computations;
2. Statements of definitions and theorems;
3. Proofs;
4. True/false with justification.

Computations. These will be drawn from computations of the type you've done on the problem sets. You do not need to explain your answer on a computational problem, but show all your work.

Statements of definitions and theorems. In these questions, you will be asked to recite a definition or the statement of a theorem from the book. You will not be asked to recite the proofs of any theorems from the book, though you may be asked to prove book theorems that you might have been asked to prove on problem sets.

Proofs. These will resemble some of the shorter problems from your homework. You may take as given anything that has been proven in class, in the homework, or in the reading. Partial credit may be given on proof questions, so keep trying if you get stuck (and you've finished everything else). If all else fails, at least try to write down the definitions of the objects involved.

True/false with justification. This type of question may be less familiar. You are given a statement, such as:

- For $a, b \in \mathbb{R}$, if $a \geq b$, then $-a \geq -b$.

If the statement is true, all you have to do is write "True". (However, see below.) If the statement is false (like the one above), not only do you have to write "False", but you must also give a reason why the statement is false. Your reason might be a very specific counterexample:

False. We have $3 \geq 2$, but $-3 < -2$, which means that $-3 \not\geq -2$.

Your reason might also be a more general principle:

False. For any $a, b \in \mathbb{R}$, if $a > b$, then $-a < -b$, which means that $-a \not\geq -b$.

Either way, your answer should be as specific as possible to ensure full credit.

Depending on the problem, some partial credit may be given if you write "False" but provide no justification, or if you write "False" but provide insufficient or incorrect justification. Partial credit may also be given if you write "True" for a false statement, but provide some partially reasonable justification. (In other words, if you have time, it can't hurt to justify "True" answers.)

If I can't tell whether you wrote "True" or "False", you will receive no credit. In particular, please do not just write "T" or "F", as you may not receive any credit.

Definitions. The most important definitions we have covered are:

Sect. 3	field	ordered field
	absolute value $|a|$	distance $|a - b|$ between a and b
Sect. 4	maximum	minimum
	closed interval $[a, b]$	open interval (a, b)
	$(a, b]$	$[a, b)$
	upper bound	bounded above
	lower bound	bounded below
	bounded	supremum
	infimum	$\mathbb{R}$ (completeness)
Sect. 5	$[a, \infty)$	(a, ∞)
	$(-\infty, b]$	$(-\infty, b)$
	$\sup S = +\infty$	$\inf S = -\infty$
Sect. 7	sequence	converge
	limit	diverge
Sect. 9	bounded (sequence)	$\lim s_n = +\infty$
	$\lim s_n = -\infty$	diverge to $\pm\infty$
Asymptotics	$a_n << b_n$	

Examples. You will also need to be familiar with the key properties of the main examples we have discussed. The most important examples we have seen are:

Sect. 4: Subsets of $\mathbb{R}$ and their maxima/minima (Ex. 1), boundedness propertes (Ex. 2), and infima/suprema (Ex. 3). Exer. 4.1: boundedness, infima/suprema.
Sect. 5: Exer. 5.1: infima/suprema.
Sect. 7: Sequences (Ex. 1) and their limits (Exs. 2 and 3). Exer. 7.3: limits.
Sect. 8: Proving limits using the definition (Exs. 1–3). Proving divergence (Ex. 4). Using $\lim s_n = s$ as an assumption (Exs. 5 and 6).
Sect. 9: $(-1)^n$ is bounded but divergent. Using limit laws (Exs. 1–3). Infinite limits (Exs. 4–7).

Theorems, results, algorithms. The most important theorems, results, and algorithms we have covered are listed below. You should understand all of these results, and you should be able to state any theorem clearly and precisely. You don't have to memorize theorems by number or page number; however, you should be able to state a theorem, given a reasonable identification of the theorem (either a name, as listed below in **boldface**, or a vague description).

Sect. 3: Field properties (Thm. 3.1), ordered field properties (Thm. 3.2), absolute value properties (Thm. 3.5). **Triangle Inequality** (Thm. 3.7).
Sect. 4: Archimedean Property (Thm. 4.6), **Density of $\mathbb{Q}$ in $\mathbb{R}$** (Thm. 4.7). (Also, though the **Completeness Axiom** is not a theorem, you should memorize it as well.)
Sect. 8: Square root limit law (Ex. 5). **Squeeze Theorem** (Exer. 8.5).
Sect. 9: Limit laws: Convergent $\Rightarrow$ bounded (Thm. 9.1), constant multiple (Thm. 9.2), sum (Thm. 9.3), product (Thm. 9.4), quotient (Thm. 9.6). Basics: $\lim(1/n^p)$ ($p > 0$), $\lim a^n$ ($|a| < 1$), $\lim n^{1/n}$, $\lim a^{1/n}$ ($a > 0$), (Thm. 9.7). $\lim s_n = +\infty$: product, reciprocal (Thms. 9.9, 9.10).
Asymptotics: Exers. 9.12, 9.14, 9.15. **The Asymptotics Theorem**: $1 << \ln n << n^t << a^n << n!$.

Not on exam. Sections 1–2 are only covered as background; no problems will refer only to those chapters.
Other. You should have a working familiarity with the techniques and strategies for proof and logic tips from the proof notes. You do not need to memorize information from the proof notes, but you do need to be able to apply it.
Good luck.