

Math 131A, problem set 09
Outline due: Wed Apr 18
Completed version due: Wed Apr 25
Last revision due: Mon May 14

Problems to be done but not turned in: 29.1, 29.3, 29.5, 29.7, 29.9, 29.11, 29.13, 29.15, 29.17, 32.1, 32.3, 32.5, 32.7, 33.1, 33.3, 33.7, 33.9, 33.11, 33.13.

Problems to be turned in: All numbers refer to exercises in Ross.

1. Prove (iv) of Corollary 29.7.
2. Let I be an interval, $a \in I$. Suppose $f : I \rightarrow \mathbf{R}$ is differentiable on I , $f'(a) > 0$, and $f'(x)$ is continuous at a .
 - (a) Prove that there exists some $\delta > 0$ such that f is strictly increasing on $[a-\delta, a+\delta]$.
 - (b) Explain why part (a) does not contradict PS08 #5(c).
3. Ex. 29.14.
4. Let $f : (0, \infty) \rightarrow \mathbf{R}$ be differentiable, and suppose that both f and f' are strictly increasing on $(0, \infty)$.
 - (a) Use the Mean Value Theorem to prove that $f'(a) > 0$ for some $a \in (1, 2)$.
 - (b) Let $m = f'(a)$, where a is taken from part (a). Prove that for $x > 2$, we have $\frac{f(x) - f(2)}{x - 2} \geq m$.
 - (c) Prove that $\lim_{x \rightarrow \infty} f(x) = +\infty$. (Suggestion: Use (b) to get a lower bound for $f(x)$.)
5. Ex. 32.6.
6. Ex. 33.8.
7. Ex. 33.10.