Math 131A, problem set 05
Outline due: Wed Mar 07
Completed version due: Mon Mar 12
Last revision due: Wed Apr 25

Problems to be done but not turned in: 12.1, 12.3, 12.5, 12.7, 12.9, 12.11, 12.13; 14.1, 14.3, 14.5, 14.7, 14.9, 14.11, 14.13.

Problems to be turned in: All numbers refer to exercises in Ross.

1. We say that a sequence (a_n, b_n) in $\mathbf{R}^2$ *converges* to a limit $(L, M) \in \mathbf{R}^2$ if $\lim a_n = L$ and $\lim b_n = M$. We also say that (a_n, b_n) is *bounded* if both (a_n) and (b_n) are bounded.

   Prove that every bounded sequence in $\mathbf{R}^2$ has a convergent subsequence. (Suggestion: Use Bolzano-Weierstrass twice, carefully.)

2. Ex. 12.12(a,b).

3. This problem proves the *Limit Comparison Test*, which is useful for the rest of this problem set.

   Let $\sum a_n$ and $\sum b_n$ be series with $a_n, b_n \geq 0$, and suppose that $\lim \dfrac{b_n}{a_n} = c$, with $0 \leq c < +\infty$ (i.e., $c \geq 0$ and $c \in \mathbf{R}$).

   (a) Prove that if $\sum a_n$ converges, then $\sum b_n$ converges. (Suggestion: Comparison test.)

   (b) Now assume that $c > 0$. Prove that if $\sum b_n$ converges, then $\sum a_n$ converges. (In other words, if $0 < c < +\infty$, then LCT is an "if and only if" condition.)

   Then converges.

4. Determine if the series $\displaystyle\sum_{n=1}^{\infty} \frac{(2n^2+13n)(n^{3-(1/n)})}{n^6 - n^2 + 7}$ converges or diverges, and prove your answer.

5. Determine if the series $\displaystyle\sum_{n=0}^{\infty} \frac{(n!)^2}{(2n)!}$ converges or diverges, and prove your answer.

6. Determine if the series $\displaystyle\sum_{n=1}^{\infty} \left(n^{(1/n)-2}\right)$ converges or diverges, and prove your answer.

7. Determine if the series $\displaystyle\sum_{n=0}^{\infty} \frac{(3^{-n}+5)1000^n}{n!}$ converges or diverges, and prove your answer.