Math 131A, problem set 03
Outline due: Wed Feb 14, 9:30am
Completed version due: Mon Feb 19
Last revision due: Mon Mar 19

Problems to be done but not turned in: 8.1, 8.3, 8.5, 8.7, 8.9, 9.1, 9.3, 9.5, 9.7, 9.9, 9.11, 9.13, 9.15, 9.17.

Problems to be turned in: All numbers refer to exercises in Ross.

1. Ex. 8.4. Note that we cannot assume that $\lim t_n$ exists.
2. Find the value of $\lim \dfrac{3 + 2\cos n^2}{n}$, and prove your answer, using either the definition of limit or the Squeeze Lemma (Ex. 8.5).
3. Suppose (s_n) is a sequence such that $\lim s_n = 7$ and $s_n \geq 7$ for all $n \in \mathbf{N}$. Let $S = \{s_n \mid n \in \mathbf{N}\}$, i.e., let S be the *set* of all values that appear in the sequence (s_n). Prove that $\inf S = 7$.
4. Suppose that (a_n) and (b_n) are sequences such that $\lim a_n = -3$ and $\lim b_n = 5$. Determine the value of $\lim \dfrac{2a_n b_n - (b_n + 1)\sqrt{7 + a_n^2}}{a_n^2 + 3}$, and carefully use the limit laws of Ch. 9 and Example 5 of Ch. 8 to prove your answer.
5. Ex. 8.8(b). Suggestion: See the square root trick of Ch. 8, and use the limit laws of Ch. 9.
6. (a) For $a \in \mathbf{R}$, suppose (x_n) is a sequence such that for all n, $|x_n - a| < \dfrac{1}{n}$. Prove that $\lim x_n = a$.
   (b) Recall that to say that S is *dense* in $\mathbf{R}$ means that for any $a, b \in \mathbf{R}$ such that $a < b$, there exists some $x \in S$ such that $a < x < b$. Prove that if S is dense in $\mathbf{R}$ and $a \in \mathbf{R}$, there exists a sequence (x_n) in S such that $a < x_n$ for all n and $\lim x_n = a$. (Suggestion: Use the previous part of the problem to choose the sequence (x_n) nonconstructively.)
7. Ex. 9.10(c).
8. Ex. 9.12.