Math 131A, problem set 01
Outline due: Wed Jan 31
Completed version due: Mon Feb 05
Last revision due: Tue Feb 27

Problems to be done but not turned in: 3.1, 3.3, 3.5, 3.7, 4.1, 4.3, 4.5, 4.7, 4.9, 4.11, 4.13, 4.15.

Problems to be turned in: All numbers refer to exercises in Ross.

1. Let a, b, c be real numbers. Using only the order axioms of $\mathbf{R}$, prove that if $a \leq b$ and $b < c$, then $a < c$. (Note that by definition, $b < c$ means $b \leq c$ and $b \neq c$. You may also find it helpful to use contradiction.)

2. Let a, b be real numbers, and suppose that for all $x < a$, we have $x \leq b$. Prove that $a \leq b$. (Suggestion: Try contradiction.)

3. For each of the following sets from Ex. 4.1, either list 3 different upper bounds **and** the sup of the set, or write NOT BOUNDED ABOVE: (e), (h), (k), (n) (the letter ℓ), (v). No proof necessary.

4. Let $S = (-3, 5)$ (an open interval). Guess the value of $\sup S$, and carefully prove your answer.

5. Let $S = \{x \in \mathbf{Q} \mid x > e\}$, where e is the base of the natural log. Guess the value of $\inf S$, and carefully prove your answer.

6. Let $S = \left\{ \dfrac{3n-5}{4n+7} \middle| n \in \mathbf{Z},\ n \geq 2 \right\}$. Guess the value of $\sup S$, and carefully prove your answer. (Do not use the idea of limit, which we have not yet defined.)

7. **(Arbitrarily Close Criterion)** Suppose S is a nonempty subset of $\mathbf{R}$, and suppose u is an upper bound for S. Prove that the following are equivalent:

   - For every $\epsilon > 0$, there exists some $s \in S$ such that $u - s < \epsilon$.
   - $u = \sup S$.

   (Suggestion: Prove that the negations of each condition are equivalent.)