Using the Limit Laws in logical order
Math 131A

In this class, when you write up answers to problems where you're supposed to apply the limit laws, I'd like you to apply the limit laws in logical order, to show your understanding of how the logic is built up.

Example: Use the limit laws to evaluate $\lim\limits_{n\to\infty} \dfrac{13n^3 + 17n^2}{5n - 12n^3}$.

Answer: First, we observe that

$$\frac{13n^3 + 17n^2}{5n - 12n^3} = \frac{\frac{13n^3}{n^3} + \frac{17n^2}{n^3}}{\frac{5n}{n^3} - \frac{12n^3}{n^3}} = \frac{13 + \frac{17}{n}}{\frac{5}{n^2} - 12}.$$

Since $\lim\limits_{n\to\infty} \dfrac{1}{n^p} = 0$ for $p > 0$, the Constant Multiple Law implies that

$$\lim_{n\to\infty} \frac{17}{n} = 0, \qquad\qquad \lim_{n\to\infty} \frac{5}{n^2} = 0.$$

(Alternatively, you can cite the Product Law and the fact that the limit of a constant is itself.)

Next, by the Sum Law and the fact that the limit of a constant is itself, we see that

$$\lim_{n\to\infty} \left(13 + \frac{17}{n}\right) = 13, \qquad\qquad \lim_{n\to\infty} \left(\frac{5}{n^2} - 12\right) = -12.$$

It then follows by the Quotient Law that

$$\lim_{n\to\infty} \frac{13n^3 + 17n^2}{5n - 12n^3} = -\frac{13}{12}.$$