Format and topics for exam 3
Math 129B

General information. Exam 3 will cover sections 6.3–6.8 and 8.1–8.3 of the text. The exam will be cumulative only to the extent that 6.3–6.8 and 8.1–8.3 rely on previous material; for example, you should still know what a subspace is and how to use the Subspace Theorem. However, there will not be any questions on the exam that only cover old material; for example, you will not be asked to recite the Subspace Theorem.

As before, most of the exam will rely on understanding the problem sets (including the problems to be done but not written up or turned in) and the definitions and theorems that lie behind them. If you can do all of the homework, and you know and understand all of the definitions and the statements of all of the theorems we've studied, you should be in good shape. You should not spend time memorizing proofs of theorems from the book, though understanding those proofs does help you understand the theorems. On the other hand, you should defintely spend time memorizing the *statements* of the important theorems in the text.

Exam 3 will follows the same ground rules as the previous exams. In particular, no books, notes, or calculators are allowed, and there will be the same four types of questions: computations, statements of definitions and theorems, proofs, and true/false with justification.

Definitions. The most important definitions we have covered are:

6.3	matrix of a linear map relative to B, B'	$[T]_{B,B'}$
6.5	change-of-basis matrix from B' to B	
6.6	kernel	null space
	column space	row space
6.7	nullity	rank
6.8	isomorphism	isomorphic
8.1	eigenvalue	eigenvector
	λ-eigenvector	λ-eigenspace
	$E_T(\lambda)$	characteristic polynomial
	characteristic equation	eigenstuff of a matrix
8.2	similar	similarity class
	characteristic polynomial (of a linear operator)	diagonal matrix
	diagonalizable	trace
8.3	basis of eigenvectors	

Examples. You will also need to be familiar with the key properties of the main examples we have discussed. Most of the important examples we have encountered have appeared in the assigned problems, both those to be turned in and those not to be turned in. In addition, you should also know:

6.6: Kernel and image of $\mu_A : \mathbb{R}^n \to \mathbb{R}^m$, differentiation, integration.
6.7: Applications of rank-nullity.
8.1: Eigenstuff for $\frac{d}{dx}, \frac{d^2}{dx^2}$ (PS08).
8.2: Matrices that have the same characteristic polynomial but are not similar.

Theorems, results, algorithms. The most important theorems, results, and algorithms we have covered are listed below. You should understand all of these results, and you should be able to state any theorem clearly and precisely. You don't have to memorize theorems by number or page number; however, you should be able to state a theorem, given a reasonable identification of the theorem (either a name or a vague description).

6.3: Every linear $T : \mathbb{R}^n \to \mathbb{R}^m$ comes from matrix multiplication; matrix of a linear transformation exists and unique; using definition of $[T]_{B,B'}$; the columns tell you where the basis goes.

6.4: Matrix of composition is product of matrices; matrix of inverse is inverse of matrix.

6.5: Change-of-basis matrix exists and is unique (and equal to $[\mathrm{id}_V]_{B',B}$); effect of change-of-basis on matrix of a linear map (the $P^{-1}AP$ formula).

6.6: Kernel and image are subspaces; $\operatorname{im}\mu_A$ is column space of A; finding bases for kernel and image of $\mu_A : \mathbb{R}^n \to \mathbb{R}^m$; finding bases for row space and column space of $A \in \mathbb{M}(m, n)$.

6.7: Dimension Theorem (rank-nullity theorem); linear map one-to-one iff kernel is 0.

6.8: Finite-dimensional vector spaces isomorphic iff same dimension.

Ch. 6 notes Whatever Theorem, SPAM Lemma, One-to-one Lemma.

8.1: Eigenvalues are roots of characteristic polynomial; computing bases for eigenspaces of a matrix.

8.2: Similarity and characteristic polynomial (Thm. 8.8).

8.3: Recognizing basis eigenvectors (Thm. 8.12); diagonalizable iff there exists a basis of eigenvectors (Thm. 8.13); The Independence of Distinct Eigenstuff (TIDE) (Thm. 8.14); Diagonalization Theorem (Thms. 8.12 + 8.13 + 8.15); linear operator with n distinct eigenvalues is diagonalizable. How to diagonalize a diagonalizable matrix: finding P, finding $P^{-1}AP$ without calculation.

Not on exam. (6.6) Row operations do not change row space; dim(row space) equals dim(col space); finding bases for row space of $A \in \mathbb{M}(m, n)$.

Good luck.