Format and topics for Exam 1
Math 129B

General information. Exam 1 will be a timed test of 75 minutes, covering 1.2–1.8 and 3.1–3.4 of the text. No books, notes, calculators, etc., are allowed. Most of the exam will rely on understanding the problem sets (including the problems to be done but not to be written up or turned in) and the definitions and theorems that lie behind them. If you can do all of the homework, and you know and understand all of the definitions and the statements of all of the theorems we've studied, you should be in good shape.

You should not spend time memorizing proofs of theorems from the book, though understanding those proofs does help you understand the theorems. On the other hand, you should defintely spend time memorizing the *statements* of the important theorems in the text.

Types of questions. There are four types of questions that may appear on exam 1, namely:

1. Computations;
2. Statements of definitions and theorems;
3. Proofs;
4. True/false with justification.

Computations. These will be drawn from computations of the type you've done on the problem sets. You do not need to explain your answer on a computational problem, but show all your work.

Statements of definitions and theorems. In these questions, you will be asked to recite a definition or the statement of a theorem from the book. You will not be asked to recite the proofs of any theorems from the book, though you may be asked to prove book theorems that you might have been asked to prove on problem sets.

Proofs. These will resemble some of the shorter problems from your homework. You may take as given anything that has been proven in class, in the homework, or in the reading. Partial credit may be given on proof questions, so keep trying if you get stuck (and you've finished everything else). If all else fails, at least try to write down the definitions of the objects involved.

True/false with justification. This type of question may be less familiar. You are given a statement, such as:

- Every subset of $\mathbb{R}^2$ is a subspace of $\mathbb{R}^2$.

If the statement is true, all you have to do is write "True". (However, see below.) If the statement is false (like the one above), not only do you have to write "False", but also, you must give a reason why the statement is false. Your reason might be a very specific counterexample:

> False. The set $S = \{(1,1)\}$ is not a subspace of $\mathbb{R}^2$, since $2(1,1) = (2,2)$ is not in S, and a subspace must be closed under scalar multiplication.

Your reason might also be a more general principle:

> False. The empty subset of $\mathbb{R}^2$ is not a subspace, since a subspace must be nonempty.

Either way, your answer should be as specific as possible to ensure full credit.

Depending on the problem, some partial credit may be given if you write "False" but provide no justification, or if you write "False" but provide insufficient or incorrect justification. Partial credit may also be given if you write "True" for a false statement, but provide some partially reasonable justification. (In other words, if you have time, it can't hurt to justify "True" answers.)

If I can't tell whether you wrote "True" or "False", you will receive no credit. In particular, please do not just write "T" or "F", as you may not receive any credit.

Definitions. The most important definitions we have covered are:

1.2	vector space	addition
	scalar multiplication	scalar
	associativity	commutativity
	additive identity	additive inverse
	closed	
1.3	uniqueness	
1.4	subtraction	
1.5	$\mathbb{R}^n$	
1.6	$\mathbb{M}(m,n)$	$[a_{ij}]$ notation
1.7	$\mathbb{F}(X)$	
1.8	subspace	closed under addition
	closed under scalar multiplication	$\mathbb{P}_n$, $\mathbb{P}$
	$\mathbb{C}(X)$	$\mathbb{D}(X)$
3.1	linear combination	coefficients (of a lin comb)
3.2	span $\{\mathbf{v}_1,\ldots,\mathbf{v}_n\}$	span (noun)
	to span (verb)	
3.3	linearly independent	linearly dependent
3.4	basis	dimension
	$\dim V$	standard basis for $\mathbb{R}^n$
	finite-dimensional	infinite-dimensional
PS02	zero function	restriction

More specifically, you should know the axioms of a vector space, but you do not need to know them by number. (Probably the best strategy is to know them by name.) You should also know the material from 1.1 and the proof notes as background.

Examples. You will also need to be familiar with the key properties of the main examples we have discussed. The most important examples we have seen are:

1.2: Zero vector space
1.5: $\mathbb{R}^n$; messed-up versions of $\mathbb{R}^n$
1.6: $\mathbb{M}(m,n)$
1.7: $\mathbb{F}(\mathbb{R})$, $\mathbb{F}(X)$
1.8: zero subspace; V as a subspace of V; polynomials (of degree $\leq n$) in $\mathbb{F}(\mathbb{R})$; $\mathbb{C}(X)$, $\mathbb{D}(X)$ in $\mathbb{F}(X)$; span $\{\mathbf{v}_1,\ldots,\mathbf{v}_n\}$; subspaces of $\mathbb{R}^2$

Theorems, results, algorithms. The most important theorems, results, and algorithms we have covered are listed below. You should understand all of these results, and you should be able to state any theorem clearly and precisely. You don't have to memorize theorems by number or page number; however, you should be able to state a theorem, given a reasonable identification of the theorem (either a name or a vague description).

1.3: Additive identity and inverse are unique; $0\mathbf{v} = \mathbf{0}$; $r\mathbf{0} = \mathbf{0}$. You will not need to recite Theorem 1.5, but you should be aware of these results and be able to use them.
1.4: Cancellation theorem. You will not need to recite Theorem 1.7, but you should be aware of these results and be able to use them.
1.8: Subspace theorem.
3.1: Using row-reduction to determine if $\mathbf{v}$ is a linear combination of $\{\mathbf{v}_1,\ldots,\mathbf{v}_n\}$.
3.2: Span is a subspace; using row-reduction to determine if $\mathbf{v}$ is in span $\{\mathbf{v}_1,\ldots,\mathbf{v}_n\}$; how to prove that $\{\mathbf{v}_1,\ldots,\mathbf{v}_n\}$ spans a vector space.
3.3: How to prove that a set of vectors is linearly independent; $\{\mathbf{v}_1,\ldots,\mathbf{v}_n\}$ linearly dependent iff one vector is a linear combination of the others.

Other. You should have a working familiarity with the techniques and strategies for proof and logic tips from the handout on "What is a proof?" You do not need to memorize information from the handout, but you do need to be able to apply it.

Good luck.