Math 129B, problem set 10
Outline due: Wed May 02
Due: Mon May 07
Last revision due: TBA

Problems to be done, but not turned in: (4.1) 4, 8, 10, 15, 17, 21; (4.4) 5, 7, 12, 15, 18.

Problems to be turned in:

1. Does
$$\langle (v_1, v_2), (w_1, w_2) \rangle = v_1 w_1 - v_1 w_2 - v_2 w_1 + v_2 w_2$$
define an inner product on $\mathbb{R}^2$? Prove or give a counterexample for each axiom that fails.

2. Does $\langle f, g \rangle = \displaystyle\int_0^1 x f(x) g(x)\, dx$ define an inner product on $\mathbb{C}([0,1])$? Prove or give a counterexample for each axiom that fails.

3. (4.1) 22.

4. Let V be an inner product space, and let S be a sub*set* (not necessarily a subspace) of V. Define
$$S^{\perp} = \{\mathbf{v} \in V \mid \langle \mathbf{u}, \mathbf{v} \rangle = 0 \text{ for all } \mathbf{u} \in S\}.$$
Prove that $S^{\perp}$ is a sub*space* of V.

5. Let V be an inner product space, and let $\mathbf{u}$ be a fixed nonzero vector in V.
   (a) Define $L_{\mathbf{u}} : V \to \mathbb{R}$ by $L_{\mathbf{u}}(\mathbf{v}) = \langle \mathbf{u}, \mathbf{v} \rangle$ for all $\mathbf{v} \in V$. Prove that $L_{\mathbf{u}}$ is linear.
   (b) What is $\operatorname{im} L_{\mathbf{u}}$? Prove your answer.
   (c) Now suppose also that V is finite-dimensional. Prove that $\dim \mathbf{u}^{\perp} = \dim V - 1$. (See problem 4 for the definition of $\mathbf{u}^{\perp}$. Suggestion: How does $\mathbf{u}^{\perp}$ relate to $L_{\mathbf{u}}$?)

6. (a) (4.4) 16(a).
   (b) (4.4) 17.