

Math 129b, problem set 01
Outline due: Wed Feb 01
Due: Mon Feb 06
Last revision due: Mon Mar 12

Problems to be done, but not turned in: (1.3) 7, 8; (1.4) 7; (1.5) 3, 5, 8, 10; (1.6) 5.

Problems to be turned in:

1. (1.3) 6. You may only rely on the axioms of a vector space and part (a) of Thm. 1.5.
2. (1.3) 11. You may only rely on the axioms of a vector space and parts (a)–(f) of Thm. 1.5.
3. (1.3) 13. You may only rely on the axioms of a vector space and (all of) Thm. 1.5.
4. Verify axiom 6 of a vector space for $\mathbb{M}(3, 2)$.
5. Let $V = \{(v_1, v_2) \mid v_1, v_2 \in \mathbb{R}\}$, and let addition and scalar multiplication in V be defined by:

$$\begin{aligned}(v_1, v_2) + (w_1, w_2) &= (v_1 + w_1 - 1, v_2 + w_2 + 3), \\ r(v_1, v_2) &= (rv_1 - r + 1, rv_2 + 3r - 3).\end{aligned}$$

Is V a vector space? If yes, verify axioms 3, 4, 5, and 7 for V . If no, find two axioms that fail, and give counterexamples to those axioms that are as specific as possible.

6. Let $V = \{(v_1, v_2) \mid v_1, v_2 \in \mathbb{R}\}$, and let addition and scalar multiplication in V be defined by:

$$\begin{aligned}(v_1, v_2) + (w_1, w_2) &= (v_2 + w_2, v_1 + w_1), \\ r(v_1, v_2) &= (rv_2, rv_1).\end{aligned}$$

Is V a vector space? If yes, verify axioms 3, 4, 5, and 7 for V . If no, find two axioms that fail, and give counterexamples to those axioms that are as specific as possible.