Math 129b, problem set 00 (review)
Due: Mon Jan 30
Last revision due: Mon Feb 13

1. Compute $\left(\begin{bmatrix} -1 & 2 \\ 0 & 3 \end{bmatrix} + 2\begin{bmatrix} 1 & 1 \\ -2 & -4 \end{bmatrix}\right)^T \begin{bmatrix} 2 \\ 3 \end{bmatrix}$.

2. Let $A = \begin{bmatrix} 3 & -6 & -2 \\ 1 & -3 & 0 \\ -2 & 1 & 3 \end{bmatrix}$ and let $B = \begin{bmatrix} 1 & 0 \\ -5 & 7 \\ 0 & 3 \end{bmatrix}$.

   (a) Choose either AB or BA, as long as the product you choose is defined, and compute its value. Show all your work.

   (b) If A is invertible, compute A^{-1}; if A is not invertible, explain how you know it is not invertible. Show all your work.

3. Find the general solution of the following system of linear equations, and put your final answer in vector form.

$$\begin{aligned} x_1 + x_2 + x_3 - x_4 &= 0, \\ 2x_2 - 2x_3 - 2x_4 &= -6, \\ -2x_1 - x_2 - 3x_3 &= -7. \end{aligned}$$

4. Let A be a 3×4 matrix whose columns are $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$, $\mathbf{v}_4$, and let $\mathbf{x} = \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix}$ be a vector in $\mathbb{R}^4$. In one sentence, describe the vector $A\mathbf{x}$ as completely as possible.

5. Given the following matrix A and the reduced row-echelon form of A:

$$A = \begin{bmatrix} 1 & 1 & 3 & 5 & -12 \\ 2 & 0 & 4 & 1 & -1 \\ 3 & -1 & 5 & 1 & -2 \end{bmatrix} \qquad \text{rref}(A) = \begin{bmatrix} 1 & 0 & 2 & 0 & 1 \\ 0 & 1 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & -3 \end{bmatrix}.$$

   (a) Let V be the column space of A. Find a basis for V, and find the dimension of V. No explanation necessary, but show all your work.

   (b) Let W be the null space of A. Find a basis for W, and find the dimension of W. No explanation necessary, but show all your work.

6. Let A be a 4×4 matrix. Explain, in one sentence, what $\det A$ tells you about A^{-1}.

In questions 7–11, you are given a statement. If the statement is true, write "True". If the statement is false, write "False", and justify your answer **as specifically as possible**.

7. If $W = \text{span}\left\{\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}\right\}$, then W contains exactly two vectors.

8. Let A, B, and C be 3×3 matrices. Then it is always the case that $(AB)C = A(BC)$.
9. Let A and B be 3×3 matrices. Then it is always the case that $AB = BA$.
10. Let A, B, and C be 3×3 matrices. Then it is always the case that $A(B + C) = AB + AC$.
11. Let $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ be vectors in $\mathbb{R}^3$ such that none of the vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ is a scalar multiple of another, i.e., $\mathbf{v}_1 \neq c\mathbf{v}_2$ for any $c \in \mathbb{R}$, $\mathbf{v}_1 \neq c\mathbf{v}_3$ for any $c \in \mathbb{R}$, and so on. Then $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ must be linearly independent.