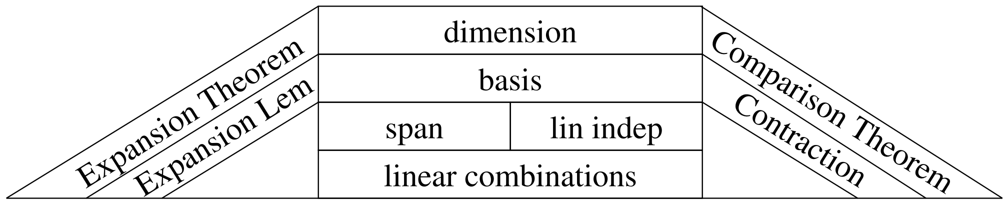
Supplemental notes on chapter 3
Math 129B

The foundations of linear algebra. As advertised:



Dimension 13. Suppose V is a vector space with $\dim V = 13$. We then know the following things about V (reasons in parentheses).

- V has some basis $\{\mathbf{v}_1, \ldots, \mathbf{v}_{13}\}$. (Definition of dimension)
- Since $\dim V > 0$, V has infinitely many different bases. (Discussed in class)
- Any basis for V has 13 vectors in it. (Comparision Thm)
- Any linearly independent set in V can contain at most 13 vectors. (Comparison Thm)
- Any set that spans V must contain at least 13 vectors. (Comparison Thm)
- Given any linearly independent set in V, we can add vectors (possibly zero of them) to obtain a basis for V. (Expansion Thm)
- Given any spanning set for V, we can remove vectors (possibly zero of them) to obtain a basis for V. (Contraction Thm)
- Any subspace W of V is finite-dimensional, with $\dim W \leq 13$. In particular, if $\dim W = 13$, then $W = V$. (Subspace Size Thm)
- If a set of 13 vectors spans V, then that set must also be linearly independent; similarly, if a set of 13 vectors in V is linearly independent, that set must also span V. (Two Out of Three Thm)

The must/may exercise. As another review of the meaning of dimension, here are 12 statements. Determine which statements are true and which are false, and give an example or justification for each answer.

Let W be a vector space such that $\dim W = 5$, and suppose that $\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4, \mathbf{u}_5, \mathbf{u}_6$ are all vectors in W.

1. It is possible that $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ spans W.
2. It is possible that $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4, \mathbf{u}_5\}$ spans W.
3. It is possible that $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4, \mathbf{u}_5, \mathbf{u}_6\}$ spans W.

4. The set $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ must span W.
5. The set $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4, \mathbf{u}_5\}$ must span W.
6. The set $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4, \mathbf{u}_5, \mathbf{u}_6\}$ must span W.
7. It is possible that $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ is linearly independent.
8. It is possible that $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4, \mathbf{u}_5\}$ is linearly independent.
9. It is possible that $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4, \mathbf{u}_5, \mathbf{u}_6\}$ is linearly independent.
10. The set $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ must be linearly independent.
11. The set $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4, \mathbf{u}_5\}$ must be linearly independent.
12. The set $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4, \mathbf{u}_5, \mathbf{u}_6\}$ must be linearly independent.