Math 129a, paragraph homework 11
More applications of eigenvalues
Due: Mon May 13

1. (5.5) 16 in textbook.

2. It is sometimes useful to think of a vector-valued function $\mathbf{y}(t) = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$ as defining a path over time in the $x - y$ plane; i.e., $(x(t), y(t))$ is where you are at time t. If $\mathbf{y}(t)$ is a solution to the system of differential equations $y' = Ay$ (where A is a 2×2 matrix), then the corresponding path is called a *solution trajectory.* The goal of this problem is to sketch the solution trajectories of a given system $\mathbf{y}' = A\mathbf{y}$.

   For the rest of the problem, let $A = \begin{bmatrix} -4 & 2 \\ -1 & -1 \end{bmatrix}$, and consider the system of differential equations $\mathbf{y}' = A\mathbf{y}$.

   (a) Find all eigenvalues of A, and find a basis for each eigenspace. (Feel free to use MATLAB to do this, though you may want to adjust the final answer, or calculate by hand, to get an integer basis.)

   (b) Find the general solution of $\mathbf{y}' = A\mathbf{y}$.

   (c) Now let $\mathbf{y}(t) = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$ be a particular example of the general solution you found in (b). As $t \to +\infty$ (i.e., as t becomes arbitrarily large and positive), what happens to $(x(t), y(t))$? Explain your answer in terms of the eigenvalues of A.

   (d) Using your answer in part (c), sketch a few solution trajectories for $y' = Ay$ on the same set of axes, using arrows to indicate the direction of increasing t (i.e., the passage of time). Your trajectories don't have to be precise; they just have to show what (c) means in terms of the picture.