Math 129a, paragraph homework 09
Basis and dimension
Due: WED APR 17

1. Let $A = \begin{bmatrix} 2 & a & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, where a is some constant, and let W be the set of all $x \in \mathcal{R}^3$ such that $A\mathbf{x} = 2\mathbf{x}$, i.e., W is the solution space to $A\mathbf{x} = 2\mathbf{x}$. You may take it as given that W is a subspace.

   Find a basis for W, and find the dimension of W, as a function of a, and describe how your answer depends on the value of a. (If you can't make sense of the general case, try some particular examples of a, e.g., $a = -1, 0, 1, \pi$, etc., and see if you can find a pattern.)

2. The solution set of a single nonzero linear equation in $\mathcal{R}^n$ is called a *hyperplane* in $\mathcal{R}^n$. For example, if $[a\ b\ c\ d]$ is a non-zero matrix, the solution set to $ax_1+bx_2+cx_3+dx_4 = 0$ in $\mathcal{R}^4$ is a hyperplane in $\mathcal{R}^4$. You may take it as given that hyperplanes are subspaces.

   (a) Suppose $[a\ b\ c\ d]$ is a non-zero matrix, and let W be the hyperplane that is the solution set to $ax_1 + bx_2 + cx_3 + dx_4 = 0$ in $\mathcal{R}^4$. Find the dimension of W, and explain your answer.

   (b) Suppose $[a\ b\ c\ d]$ and $[a'\ b'\ c'\ d']$ are non-zero matrices, and let

   $$\begin{aligned} W &= \text{solution set to } ax_1 + bx_2 + cx_3 + dx_4 = 0, \\ W' &= \text{solution set to } a'x_1 + b'x_2 + c'x_3 + d'x_4 = 0. \end{aligned}$$

   The *intersection* of the hyperplanes W and W' is the set of all $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$ that are solutions to both $ax_1 + bx_2 + cx_3 + dx_4 = 0$ and $a'x_1 + b'x_2 + c'x_3 + d'x_4 = 0$. Let V be the intersection of W and W'. Taking it as given that V is a subspace of $\mathcal{R}^4$, what are the possible values of the dimension of V? Explain. (I.e., your answer should be something like, "The dimension of V could be either 4 or 7, because . . . ")