Math 129a, paragraph homework 04
Linear dependence and independence
Due: Fri Feb 22

1. Consider the following homogenous (see p. 72) system of linear equations in six variables:

$$\left[\begin{array}{cccccc|c} 1 & 0 & -2 & 0 & 1 & -4 & 0 \\ 0 & 1 & 3 & 0 & -7 & 6 & 0 \\ 0 & 0 & 0 & 1 & 5 & -11 & 0 \end{array}\right]$$

   Note that this system is in RREF.

   (a) Write down the general solution to this system in vector form. (This is also known as the parametric representation of the general solution to the system; see p. 73.)

   (b) Note that the vector form of the general solution expresses the solution set as the span of three vectors. **Explain** why these three vectors are linearly independent, without doing any computations (i.e., just using the definition of linear independence).

2. Suppose that $\mathbf{u}$, $\mathbf{v}$, $\mathbf{w}$, and $\mathbf{x}$ are vectors in $\mathcal{R}^5$ such that $2\mathbf{u} + \mathbf{v} - 7\mathbf{w} + 6\mathbf{x} = \mathbf{0}$. Nothing else is known about $\mathbf{u}$, $\mathbf{v}$, $\mathbf{w}$, and $\mathbf{x}$.

   (a) **Must** it be true that $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly dependent? If yes, carefully **explain** why, using the definition of linear dependence; if no, give one example of vectors $\mathbf{u}, \mathbf{v}, \mathbf{w}, \mathbf{x} \in \mathcal{R}^5$ such that $2\mathbf{u} + \mathbf{v} - 7\mathbf{w} + 6\mathbf{x} = \mathbf{0}$ and $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly *in*dependent.

   (b) Now let $\mathbf{y}$ be another vector in $\mathcal{R}^5$. **Must** it be true that $\{\mathbf{u}, \mathbf{v}, \mathbf{w}, \mathbf{x}, \mathbf{y}\}$ is linearly dependent? If yes, carefully **explain** why, using the definition of linear dependence; if no, give one example of vectors $\mathbf{u}, \mathbf{v}, \mathbf{w}, \mathbf{x}, \mathbf{y} \in \mathcal{R}^5$ such that $2\mathbf{u} + \mathbf{v} - 7\mathbf{w} + 6\mathbf{x} = \mathbf{0}$ and $\{\mathbf{u}, \mathbf{v}, \mathbf{w}, \mathbf{x}, \mathbf{y}\}$ is linearly *in*dependent.