Format and topics
Final exam, Math 128B

General information. The final will be somewhat less than twice as long as our in-class exams, with 135 minutes in which to complete it, and it will take place in our usual room.

The final will be **cumulative**; in other words, the final will cover the topics on this sheet and the topics on the previous three review sheets. However, the exam will somehwat emphasize the material listed here from Chapters 24 and 32.

As always, most of the exam will rely on understanding the problem sets and the definitions and theorems that lie behind them. If you can do all of the homework, and you know and understand all of the definitions and the statements of all of the theorems we've studied, you should be in good shape. You should not spend time memorizing proofs of theorems from the book, though understanding those proofs may help you understand the theorems. On the other hand, you should defintely spend time memorizing the *statements* of the important theorems in the text.

As usual, no books or notes allowed, and four basic types of questions: computations, statements of definitions and theorems, proofs, and true/false with justification. (Remember to be as specific as possible on the true/false questions.)

Definitions. The most important new definitions we have covered are:

Ch. 24	conjugate (elements)	conjugacy class
	p-group	Sylow p-subgroup
	conjugate (subgroups)	
Actions	group action	orbit
	stabilizer	
Ch. 32	automorphism (of a field)	Galois group of E over F
	$\mathrm{Gal}(E/F)$	fixed field
	solvable by radicals over F	solvable (group)

Examples. You will also need to be familiar with the key properties of the main examples we have discussed. The most important new examples we have seen are:

Ch. 24 Conjugacy classes of D_4. Groups of order 40, 30, 99, 66, 255.

Actions S_n acting on $\{1, \dots, n\}$; D_n acting on regular n-gon. G acting by left translation on its elements (left regular represenation); G acting by conjugacy on elements and subgroups.

Ch. 32 Subfields of $\mathbf{Q}(\sqrt[4]{2}, i)$ vs. subgroups of D_4. $\mathrm{Gal}(E/\mathbf{Q})$ and subfields of E for $E = \mathbf{Q}(\sqrt{2})$, $\mathbf{Q}(\sqrt[3]{2})$, $\mathbf{Q}(\sqrt[4]{2}, i)$, $\mathbf{Q}(\sqrt{3}, \sqrt{5})$, $\mathbf{Q}(\omega_3, \sqrt[3]{2})$, $\mathbf{Q}(\omega_7)$ ($\omega_n = e^{2\pi i/n}$). $\mathrm{Gal}(\mathbf{Q}(\sqrt[4]{2}, i)/\mathbf{Q}(i))$ and subfields. $\mathrm{Gal}(\mathrm{GF}(p^n)/\mathrm{GF}(p))$ and subfields of $\mathrm{GF}(p^n)$. $x^n - a$ solvable by radicals. $3x^5 - 15x + 5$ and $x^5 - 4x - 2$ not solvable by radicals.

Theorems, results, algorithms. The most important new theorems, results, and algorithms we have covered are listed below. You should understand all of these results, and you should be able to state any theorem clearly and precisely. You don't have to memorize theorems by number or page number; however, you should be able to state a theorem, given a reasonable identification of the theorem (either a name or a vague description).

Ch. 24 Orbit-Stabilizer for conjugacy classes (Thm. 24.1), the class equation. p-groups have $Z(G) \neq \{e\}$, groups of order p^2 are abelian. Sylow Theorem I (existence), Sylow Theorem II (containment in Sylow p-subgroups), Sylow Theorem III (number of Sylow p-subgroups, all are conjugate). Groups of order pq are cyclic for $p < q$, $q \not\equiv 1 \pmod{p}$.

Actions Orbit-Stabilizer Theorem.

Ch. 32 Fundamental Theorem of Galois Theory. Galois group of the splitting field of $x^n - a$ is solvable (Thm. 32.2); factor group of solvable is solvable (Thm. 32.3), N and G/N solvable implies G solvable (Thm. 32.4); Solvable by radicals implies solvable group (Thm. 32.5).

Not on exam. (Ch. 24) Probability that two elements commute.

Good luck.