Format and topics
Exam 3, Math 128B

General information. Exam 3 will be a timed test of 75 minutes, covering Chapters 20–23 of the text. You are allowed

ONE PAGE OF NOTES

and no other aids (books or calculators).

As before, most of the exam will rely on understanding the problem sets and the definitions and theorems that lie behind them. If you can do all of the homework, and you know and understand all of the definitions and the statements of all of the theorems we've studied, you should be in good shape. You should not spend time memorizing proofs of theorems from the book, though understanding those proofs does help you understand the theorems. On the other hand, you should defintely spend time memorizing the *statements* of the important theorems in the text.

The exam will contain the same four types of questions as the previous one. (Remember to be as specific as possible on the true/false questions.) The exam will not be cumulative, per se, as there will not be any questions that only concern material before Ch. 20. However, it will be assumed that you still understand the previous material; for example, it will be assumed that you know what rings and ideals are, what the characteristic of a ring is, what $\mathbf{R}[x]$ is, and so on.

Definitions. The most important definitions we have covered are:

Ch. 20	extension field	splitting field for $f(x)$ over E
	$F(a_1, \ldots, a_n)$	derivative
	multiple zeros	perfect field
	$F(\alpha)$ where α is a root of $f(x)$	
Ch. 21	algebraic (field element) over F	transcendental (field element) over F
	algebraic extension	transcendental extension
	simple extension	minimal polynomial for a over F
	degree (of an extension field)	$[E : F]$
	finite extension	infinite extension
	degree (of a field element)	primitive element
	algebraic closure of F in E	algebraically closed
	algebraic closure of F	
Ch. 22	$GF(p^n)$	Galois field of order p^n
Ch. 23	constructible number	plane of F
	line in F	circle in F

Examples. You will also need to be familiar with the key properties of the main examples we have discussed. The most important examples we have seen are:

Ch. 20 Adjoining elements to find a zero for $f(x)$ (Exmps. 1, 2, 7). Examples of splitting fields (Exmps. 4–6, 8; PS07 and other problems).

Ch. 21 Computing $[\mathbf{Q}(i) : \mathbf{Q}]$, $[F(a) : F]$. Extensions: $\mathbf{Q}(\sqrt{3}, \sqrt{5})$, $\mathbf{Q}(\sqrt[3]{2}, \sqrt[4]{2})$ (Exmps. 3, 4, 6, 7). Showing that an element is not in a particular field (Exmp. 5).

Ch. 22 GF(16): list of elements, how to compute, log/antilog tables (Exmp. 1, PS09). Factoring $x^3 + x^2 + 1$ and $x^3 + x + 1$ in their splitting fields over $\mathbf{Z}_2$. $x + \langle f(x) \rangle$ is not always a generator of the multiplicative group of $\mathbf{Z}_p[x]/\langle f(x) \rangle$ (Exer. 17).

Theorems, results, algorithms. The most important theorems, results, and algorithms we have covered are listed below. You should understand all of these results, and you should be able to state any theorem clearly and precisely. You don't have to memorize theorems by number or page number; however, you should be able to state a theorem, given a reasonable identification of the theorem (either a name or a vague description).

Ch. 20 Fundamental Theorem of Field Theory. Every $f(x)$ has a splitting field. For $p(x)$ irreducible, $F(a) \approx F[x]/\langle p(x)\rangle \approx F(b)$. Splitting fields are unique. Properties of the derivative. Derivative criterion for multiple zeros. Finite fields are perfect. Irreducible polynomials over perfect fields do not have multiple zeros (Thm. 20.8).

Ch. 21 **The Minimal Polynomial Theorem:** Includes Thms. 21.1–21.3. Finite implies algebraic (Thm. 21.4), Multiplicativity of Degree (Thm. 21.5). Primitive Element Theorem (Thm. 21.6). Algebraic over Algebraic Is Algebraic (Thm. 21.7), Algebraic elements form a subfield.

Ch. 22 Classification of finite fields. Additive and multiplicative structure of a finite field (Thm. 22.2). $[\mathrm{GF}(p^n) : \mathrm{GF}(p)] = n$. Subfields of a finite field (Thm. 22.3).

Ch. 23 Characterization of constructible real numbers (p. 380).

Not on exam. (Ch. 20) Thm. 20.6; Thm. 20.9 and Corollary.

Good luck.