Format and topics
Exam 1, Math 128B

General information. Exam 1 will be a timed test of 75 minutes, covering Chapters 12–16 of the text. No books, notes, calculators, etc., are allowed. Most of the exam will rely on understanding the problem sets and the definitions and theorems that lie behind them. If you can do all of the homework, and you know and understand all of the definitions and the statements of all of the theorems we've studied, you should be in good shape.

You should not spend time memorizing proofs of theorems from the book, though understanding those proofs does help you understand the theorems. On the other hand, you should defintely spend time memorizing the *statements* of the important theorems in the text.

Types of questions. There are four types of questions that may appear on exams in this class, namely: computations; statements of definitions and theorems; proofs; and true/false with justification.

Computations. These will be drawn from computations of the type you've done on the problem sets. You do not need to explain your answer on a computational problem, but show all your work.

Statements of definitions and theorems. In these questions, you will be asked to recite a definition or the statement of a theorem from the book. You will not be asked to recite the proofs of any theorems from the book, though you may be asked to prove book theorems that you might have been asked to prove on problem sets.

Proofs. These will resemble some of the shorter problems from your homework. You may take as given anything that has been proven in class, in the homework, or in the reading. Partial credit may be given on proof questions, so keep trying if you get stuck (and you've finished everything else). If all else fails, at least try to write down the definitions of the objects involved.

True/false with justification. This type of question may be less familiar. You are given a statement, such as:

- If G is a group, with its operation written multiplicatively, and $a, b \in G$, then $(ab)^{-1} = b^{-1}$.

If the statement is true, all you have to do is write "True". (However, see below.) If the statement is false (like the one above), not only do you have to write "False", but you must also give a reason why the statement is false. Your reason might be a very specific counterexample:

> False. For $G = \mathbf{R}^*$ (the nonzero real numbers), $a = 2$, and $b = 3$, $(ab)^{-1} = 1/6$, but $b^{-1} = 1/3$.

Your reason might also be a more general principle:

> False. In that case, we would have $ab = b$, which by cancellation means that $a = e$. So the statement fails for any $a \neq e$.

Either way, your answer should be as specific as possible to ensure full credit.

Depending on the problem, some partial credit may be given if you write "False" but provide no justification, or if you write "False" but provide insufficient or incorrect justification. Partial credit may also be given if you write "True" for a false statement, but provide some partially reasonable justification. (In other words, if you have time, it can't hurt to justify "True" answers.)

If I can't tell whether you wrote "True" or "False", you will receive no credit. In particular, please do not just write "T" or "F", as you may not receive any credit.

Definitions. The most important definitions we have covered are:

Ch. 12	ring	commutative ring
	unity, 1	unit, a^{-1}
	divide, factor, $a \mid b$	$R[x]$ (R a comm. ring)
	$M_n(R)$ (R a comm. ring)	direct sum $R_1 \oplus \cdots \oplus R_n$
	subring	Gaussian integers $\mathbf{Z}[i]$
	$n\mathbf{Z}$	

Ch. 13	zero-divisor	integral domain
	field	characteristic of a ring
	char R	
Ch. 14	ideal	proper ideal
	$\langle a_1, \ldots, a_n \rangle$	ideal generated by $a_1, \ldots, a_n$
	factor ring R/A	prime ideal
	maximal ideal	
Ch. 15	ring homomorphism	ring isomorphism
	kernel	$\ker \varphi$
	natural homomorphism $R \to R/A$	prime subfield
	field of quotients	$F(x)$ (quotients of $F[x]$)

Examples. You will also need to be familiar with the key properties of the main examples we have discussed. The most important examples we have seen are:

Ch. 12 $\mathbf{Z}$, $\mathbf{Q}$, $\mathbf{C}$, $\mathbf{R}$, $\mathbf{Z}_n$, $R[x]$, $M_n(R)$, $n\mathbf{Z}$, real-valued functions, $\mathbf{Z}[i]$, $R_1 \oplus \cdots \oplus R_n$. Trivial subring, diagonal matrices, upper triangular matrices.

Ch. 13 Integral domains: $\mathbf{Z}$, $\mathbf{Z}[i]$, $R[x]$ (R is an integral domain), $\mathbf{Z}_p$. Non-integral domains: $\mathbf{Z}_n$ (n composite), $R_1 \oplus R_2$. Fields: $\mathbf{Z}_3[i]$, $\mathbf{Q}(\sqrt{2})$.

Ch. 14 Ideals: R and $\{0\}$ in R, $n\mathbf{Z}$ in $\mathbf{Z}$, $\langle a_1, \ldots, a_n \rangle$ in R (examples of this class). Non-ideals: even-degree polynomials in $R[x]$. Factor rings: $\mathbf{Z}/n\mathbf{Z}$, $\mathbf{Z}[i]/\langle 2-i \rangle$, $\mathbf{R}[x]/\langle x^2+1 \rangle$. Prime and maximal ideals of $\mathbf{Z}$, $\mathbf{Z}_{36}$; $\langle x^2+1 \rangle$ maximal in $\mathbf{R}[x]$, $\langle x^2+1 \rangle$ not prime in $\mathbf{Z}_2[x]$.

Ch. 15 Natural homomorphism $\mathbf{Z} \to \mathbf{Z}_n$; ring automorphisms $a+bi \mapsto a-bi$, $a+b\sqrt{2} \mapsto a-b\sqrt{2}$; substitution homomorphism $p(x) \mapsto p(a)$. All ring homomorphisms $\mathbf{Z}_m \to \mathbf{Z}_n$. Applications: Divisibility by 9, reduction mod 8.

Theorems, results, algorithms. The most important theorems, results, and algorithms we have covered are listed below. You should understand all of these results, and you should be able to state any theorem clearly and precisely. You don't have to memorize theorems by number or page number; however, you should be able to state a theorem, given a reasonable identification of the theorem (either a name or a vague description).

Ch. 12 Rules of multiplication. Unity and inverses unique. Subring Test.

Ch. 13 Cancellation. Finite integral domains are fields; $\mathbf{Z}_p$ is a field.

Ch. 14 Ideal test. Factor rings well-defined. R/A is an integral domain iff A is prime; $\mathbf{R}/A$ is a field iff A is maximal.

Ch. 15 Basic properties of ring homomorphisms (Thm. 15.1). Kernels are ideals and vice versa. First Isomorphism Theorem for rings. Homomorphism from $\mathbf{Z}$ to ring with unity; corollaries: ring with unity contains $\mathbf{Z}_n$ or $\mathbf{Z}$, $\mathbf{Z}_m$ is a homomorphic image of $\mathbf{Z}$, a field contains $\mathbf{Z}_p$ or $\mathbf{Q}$. Field of quotients is a well-defined field.

Good luck.