

Math 128B, problem set 01
Outline due: Wed Feb 03
Due: Mon Feb 08
Last revision due: Mon Mar 01

Problems to be done, but not turned in: (Ch. 12) 1–57 odd. (Ch. 13) 1–27 odd.

Problems to be turned in:

1. Let R be the set $\mathbf{R} \cup \{\infty\}$ (i.e., the set of all real numbers along with the symbol ∞), with operations \oplus and \otimes given by

$$a \oplus b = \min(a, b),$$
$$a \otimes b = a + b,$$

for all $a, b \in R$, where we define $\min(a, \infty) = \min(\infty, a) = a$ and $a + \infty = \infty + a = \infty$ for all $a \in R$.

Prove that R , with the operations of \oplus and \otimes , satisfies five of the six axioms of a ring. More precisely, for each axiom that R satisfies, prove that R satisfies that axiom; and for each axiom that R does not satisfy, give a specific counterexample that shows that R does not satisfy the axiom.

2. (Ch. 12) 6.
3. (Ch. 12) 12.
4. (Ch. 12) 32.
5. (Ch. 12) 46.
6. (Ch. 13) 18.
7. Describe all zero-divisors, units, and idempotents of $\mathbf{Z} \oplus \mathbf{R}$.