Math 128B, problem set 01
Outline due: Wed Feb 02
Due: Mon Feb 07
Last revision due: Mon Feb 21

Problems to be done, but not turned in: (Ch. 12) 1, 3, 9, 13, 15, 17, 19, 25, 41, 49. (Ch. 13) 3, 9, 13, 25.
Fun: (Ch. 12) 33.

Problems to be turned in:

1. Let R be the set $\mathbf{R} \cup \{\infty\}$ (i.e., the set of all real numbers along with the symbol ∞), with operations $\oplus$ and $\otimes$ given by

$$a \oplus b = \min(a, b),$$
$$a \otimes b = a + b,$$

   for all $a, b \in R$, where we define $\min(a, \infty) = \min(\infty, a) = a$ and $a + \infty = \infty + a = \infty$ for all $a \in R$.

   Prove that R, with the operations of $\oplus$ and $\otimes$, satisfies five of the six axioms of a ring. More precisely, for each axiom that R satisfies, prove that R satisfies that axiom; and for each axiom that R does not satisfy, give a specific counterexample that shows that R does not satisfy the axiom.

2. (Ch. 12) 4.

3. (Ch. 12) 12.

4. (Ch. 12) 18.

5. (Ch. 12) 48. Prove your answer.

6. (Ch. 13) 16.

7. Describe all zero-divisors, units, and idempotents of $\mathbf{Z} \oplus \mathbf{Q}$.