Math 128A, Mon Dec 07

- Use a laptop or desktop with a large screen so you can read these words clearly.
- In general, please turn off your camera and mute yourself.
- Exception: When we do groupwork, please turn both your camera and mic on. (Groupwork will not be recorded.)
- Please always have the chat window open to ask questions.
- PS11 due tonight.
- All revisions due **Fri Dec 18**.
- Final exam review, **Fri Dec 11, 9:45am–noon.**
- **FINAL EXAM, TUE DEC 15, 7:15–9:30am.**

Chapter 1: Dihedral groups D_n

Ch. 0:
* Partitions & equivalence relations
* One-to-one and onto

(Ch. 0: GCD is a linear combination)

- Definition; cyclic groups C_n
- Review notation and computation; have n-gons handy

$C_n = \langle R_{\frac{360}{n}} \rangle$

Chapter 2

Groups!

- Definition of group

$\mathbb{Z}, GL(2, \mathbb{R})$

- Many, many examples: For each group example, know notation, elements, operation, identity, inverses
- First properties (e.g., uniquenesses, cancellation, socks and shoes)

$$(ab)^{-1} = b^{-1}a^{-1}$$

Chapter 3: Subgroups

- Orders of groups and orders of elements
- Defn of subgroup
- Subgroup tests

1-step
2-step

$\boxed{H \neq \emptyset}$

- Examples of subgroups and how to use tests

$a \in G$ (multiplicative)

$\text{ord}(a) = \boxed{\text{smallest}}\ n > 0$ s.t. $a^n = e$

Chapter 4: Cyclic Groups

(Recall: $a^n = e \iff \mathrm{ord}(a)$ divides n)

- How to compute in cyclic groups (Thm. 4.1 and corollaries)
- Orders of elements and generators(Thm. 4.2 and corollaries)
- Fundamental Theorem of Cyclic Groups
- Number of elements of a given order *d* (for both cyclic groups and groups in general)

If $\mathrm{ord}(a) = n$, $\mathrm{ord}(a^k) = \dfrac{n}{\gcd(n,k)}$

If G cyclic, and d divides |G|: There are *exactly* $\varphi(d)$ elts of order d.

In general, if d divides |G|: # of elts of order d is a *multiple* of $\varphi(d)$.

Chapter 5: Permutation Groups

- Definition as bijections on $\{1, \ldots, n\}$; Symmetric group S_n
- Two-line and disjoint cycle notation
- Multiplying permutations **right to left**
- Disjoint cycles
- Order of a permutation
- Even and odd permutations
- Alternating group A_n

$$(1\,2\,3) = (1\,2)(2\,3)$$

$$2 \leftarrow 1 \leftarrow 1$$
$$3 \leftarrow 3 \leftarrow 2$$
$$1 \leftarrow 2 \leftarrow 3$$

Even and odd permutations vs. permutations of even and odd order:

$$\underbrace{(1\;2\;3\;\cdots\;k)}_{k\text{-cycle, order } k} = \underbrace{(1\,2)(2\,3)\cdots(k{-}1\;k)}_{(k-1)\ 2\text{-cycles}}$$

So:

A cycle of ODD length is an EVEN permutation.
A cycle of EVEN length is an ODD permutation.

If σ is expressed as a product of disjoint cycles, then σ is an EVEN permutation if and only if it its disjoint cycle form has an EVEN number of cycles of EVEN length.

E.g., (1 2 3 4)(5 6 7)(8 9) is an even permutation, but (1 2 3 4)(5 6)(7 8) is an odd permutation.

Every permutation of ODD order is a product of cycles of ODD length, and is therefore an EVEN permutation; but permutations of EVEN order are sometimes ODD permutations and sometimes EVEN permutations.

Mult table for perms

	E	O
E	E	O
O	O	E

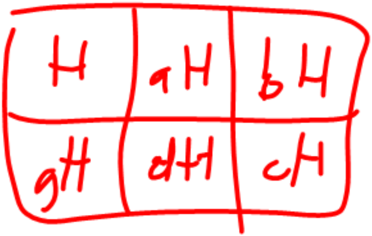
Even
Odd

Chapter 6: Isomorphisms

- When are two groups "the same"?
- Definition of isomorphism, isomorphic
- Examples
- Cayley's Thm
- Properties preserved under isomorphism (element-wise and subgroup-wise)
- Automorphisms: Aut(G), Inn(G)

Chapter 7: Cosets and Lagrange's Thm

- Definition of cosets
- Properties of cosets (when is $aH = bH$? when is $a + H = b + H$?)
- Lagrange's Thm and corollaries
- Groups of prime order p
- $|HK| = |H|\,|K|\,/\,|H \cap K|$
- Groups of order $2p$
- Orbit-Stabilizer



Frequent problem: Prove that a group of a given order must have elements of some given order.

Chapter 8: External Direct Products

- Defn of $G \oplus H$
- Examples
- Order of an element in $G \oplus H$
- When is $\mathbf{Z}_n \oplus \mathbf{Z}_k$ cyclic?
- $U(n)$ as an external direct product

Chapter 9: Normal Subgroups and Factor Groups

- Defn of normal subgroups
- Normal Subgroup Test
- Factor groups: Elements, operation, identity, inverse
- Examples and applications
- Internal direct product (when is $G \approx H \oplus K$?)
- Groups of order p^2

Chapter 10: Homomorphisms

If $ab = ba$

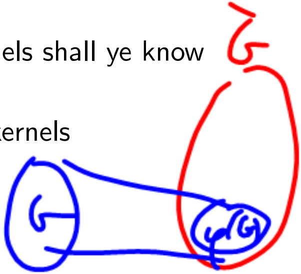
$\Rightarrow \varphi(a)\varphi(b) = \varphi(b)\varphi(a)$

$G_{ab} \Rightarrow \varphi(G)_{ab}$

- Defn of homomorphism
- Defn of kernel
- Examples
- Properties preserved under homomorphism and preimage (element-wise and subgroup-wise)
- First Isomorphism Theorem ("By their kernels shall ye know them"); $|\ker\varphi|$-to-1 maps
- Kernels are normal; normal subgroups are kernels

$\varphi : G \to \overline{G}$ $\quad T = \ker\varphi$

1IT $\quad G/T \approx \varphi(G)$



Chapter 11: Classification of Finite Abelian Groups

- Statement of the classification
- Abelian groups of a given order n
- Identifying a given finite abelian group

$$\begin{array}{ccccc} \mathbb{Z}_8 & \oplus & \mathbb{Z}_{25} & \oplus & \mathbb{Z}_7 \\ \mathbb{Z}_4 \oplus \mathbb{Z}_2 & & \mathbb{Z}_5 \oplus \mathbb{Z}_5 & & \\ \mathbb{Z}_2 \oplus \mathbb{Z}_2 \oplus \mathbb{Z}_2 & & & & \end{array}$$

Something from Col 1 x something from Col 2 x something from Col 3

Chapter 12: Rings

- Defn of ring
- Examples
- First properties of rings (e.g., uniquenesses)
- Subrings: Defns
- Subring Test

Chapter 14: Factor rings

- Defn of ideal
- Ideal Test
- Defn of factor ring: Elements, operations, zero

R/A

elt $= 0+A$
$= A$

See: PS 10-11

Ex. $G = \mathbb{Z}_2 \oplus \mathbb{Z}_8$

$N = \langle (1,2) \rangle = \left\{ \begin{matrix} (0,0), (1,2) \\ (0,4), (1,6) \end{matrix} \right\}$

Find ord of $(0,1)+N$ in G/N. 4

Ans ord = smallest n s.t. $n \cdot ((0,1)+N) = N$

$1 \cdot ((0,1)+N) = (0,1)+N \neq N$ ← $(0,1) \notin N$

$2 \cdot ((0,1)+N) = (0,2)+N \neq N$

$3 \cdot ((0,1)+N) = (0,3)+N \neq N$

$4 \cdot ((0,1)+N) = (0,4)+N = N$ b/c $(0,4) \in N$