Sample Exam 3
Math 128A, Fall 2010

This test covered Chs. 8–12, not 7–11, so expect the Ch. 12 material to be replaced by material from Ch. 7.

1. (12 points) (From Ch. 12) Define what it means for R to be a ring.

2. (10 points)

(a) **How many** abelian groups of order $11^3 17^5$ are there, up to isomorphism? Justify your answer. (You do not need to list all such groups; just explain how you can be sure that your count is correct.)

(b) List **one** abelian group of order $11^3 17^5$ that is **not** cyclic. No explanation necessary.

For questions 3–6, you are given a statement. If the statement is true, you need only write "True", though a justification may earn you partial credit if the correct answer is "False". If the statement is false, write "False", and justify your answer **as specifically as possible**. (Do not just write "T" or "F", as you may not receive any credit; write out the entire word "True" or "False".)

3. (12 points) Let G be a group, and let H be a subgroup of G. If G is abelian, then it must be the case that H is normal in G.

4. (12 points) Let $\mathbf{R}^*$ be the multiplicative group of the nonzero reals. Then the function $\varphi : \mathbf{R}^* \to \mathbf{R}^*$ given by $\varphi(x) = 2x$ is a homomorphism.

5. (12 points) Let G and $\overline{G}$ be groups, let $\varphi : G \to \overline{G}$ be a homomorphism, and suppose $g \in G$ and $\overline{g} \in \overline{G}$. If $\varphi(g) = \overline{g}$, then it must be the case that $\varphi^{-1}(\overline{g}) = \{g\}$.

6. (12 points) Let G and H be groups, and suppose that for some $(a, b) \in G \oplus H$, (a, b) has order 6. Then it must be the case that $|a|\,|b| = 6$.

7. (14 points) **PROOF QUESTION.** Let G be a group, let N be a normal subgroup of G, and suppose that for all $a \in G$ there exists an integer $k > 0$ such that $a^k \in N$. Prove that for all $x \in G/N$, x has finite order.

8. (16 points) **PROOF QUESTION.** Let G be a group with identity e and let $\overline{G}$ be a group with identity $\overline{e}$; let $\varphi : G \to \overline{G}$ be a surjective (onto) homomorphism; and let

$$K = \{g \in G \mid \varphi(g) = \overline{e}\}.$$

Suppose also that $\overline{G}$ is abelian. Prove that for all $a, b \in G$, $abK = baK$.