

Sample Exam 2
Math 128A, Fall 2010

1. (12 points) For $i = 1, 2, 3$, let G_i be a group with a multiplicative operation and identity e_i . Define the external direct product $G_1 \oplus G_2 \oplus G_3$. In particular, define the elements and the operation of $G_1 \oplus G_2 \oplus G_3$, and describe the identity and inverses in $G_1 \oplus G_2 \oplus G_3$.

2. (10 points) Let

$$\alpha = (1\ 8)(2\ 5\ 4\ 6)(3\ 9\ 7), \quad \beta = (2\ 3\ 8\ 9\ 5\ 6)$$

Compute the permutation $\alpha\beta$, and find the orders of α , β , and $\alpha\beta$. No explanation necessary, but show all your work.

For questions 3–6, you are given a statement. If the statement is true, you need only write “True”, though a justification may earn you partial credit if the correct answer is “False”. If the statement is false, write “False”, and justify your answer **as specifically as possible**. (Do not just write “T” or “F”, as you may not receive any credit; write out the entire word “True” or “False”.)

3. (12 points) For $\alpha, \beta \in S_{23}$, it is possible that $\alpha\beta\alpha^{-1}\beta^{-1}$ is an odd permutation.

4. (12 points) Suppose G is a group of order 78 and H is a group containing an element of order 5. It is possible that G is isomorphic to H .

5. (12 points) If G is a group, H is a subgroup of G , and a and b are elements of G such that $a \neq b$, then it must be the case that $aH \neq bH$.

6. (12 points) If G and H are groups such that G is isomorphic to S_6 and H is isomorphic to S_6 , then it must be the case that G is isomorphic to H .

7. (13 points) **PROOF QUESTION.** Let H be a subgroup of S_{10} such that

$$\alpha = (1\ 5)(2\ 9\ 6\ 3\ 4)(7\ 8\ 10)$$

is an element of H . Prove that there exist at least ten (10) elements of H that satisfy the equation $x^{10} = e$.

8. (17 points) **PROOF QUESTION.** Let G be a group of order 77. Prove that G contains at least one element of order 7 and at least one element of order 11. (Suggestion: You may want to divide your proof into the case where G is cyclic and the case where G is not cyclic.)