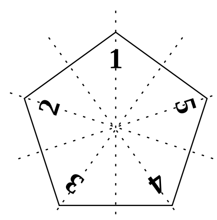
Sample Exam 1
Math 128A, Fall 2010

1. (12 points) State the Fundamental Theorem of Cyclic Groups. (This is the theorem that gives a precise description of all subgroups of a cyclic group $G = \langle a \rangle$, especially when G is finite.)

2. (10 points) Recall that in the group D_5, R_d is a counterclockwise rotation of d degrees, and for $1 \le i \le 5$, F_i is the reflection across the line shown below.



Compute $F_3 R_{144}$. Show all your work; in particular, show the results of any intermediate steps.

For questions 3–6, you are given a statement. If the statement is true, you need only write "True", though a justification may earn you partial credit if the correct answer is "False". If the statement is false, write "False", and justify your answer **as specifically as possible**. (Do not just write "T" or "F", as you may not receive any credit; write out the entire word "True" or "False".)

3. (12 points) Let G be a group, and let x be an element of G. If $x^4 = e$, then it must be the case that the order of x is 4.

4. (12 points) Let G be a group, and let a, b, c be elements of G. If $b = c$, then it must be the case that $ab = ca$.

5. (12 points) For every integer k such that $1 \le k \le 12$, there exists $x \in \mathbf{Z}$ such that $kx = 1 \pmod{13}$.

6. (12 points) Let G be a group. If H is a nonempty subset of G, then it must be the case that H is a subgroup of G.

7. (12 points) **PROOF QUESTION.** Let a, b, and c be positive integers, and let $d = \gcd(a, b)$. Prove that there exist $x, y, z \in \mathbf{Z}$ such that $ax + by + cz = \gcd(d, c)$.

8. (18 points) **PROOF QUESTION.** Let H be a subgroup of $GL(2, \mathbf{R})$, and let

$$\begin{aligned} L &= \{x \in \mathbf{R}^* \mid x = \det A \text{ for some } A \in H\} \\ &= \{\det A \mid A \in H\}. \end{aligned}$$

Prove that L is a subgroup of $\mathbf{R}^*$ (the multiplicative group of the nonzero reals).