Sample Exam 2
Math 128A, Fall 2020

Note that this exam began with a "recite the definition" problem, which is a type of problem that your exam will not have. On the other hand, there will most probably be some kind of computational problem where it will be helpful to know definitions well, so either memorizing or carefully writing down definitions in your notes will be helpful.

1. (10 points) Let G and H be groups.

(a) Define what it means for $\varphi : G \to H$ to be an isomorphism.

(b) Define what it means for G and H to be isomorphic.

2. (10 points) Let $H = \langle 15 \rangle$ be the subgroup of $\mathbf{Z}_{20}$ generated by the element 15. List all of the subgroups of H. (When you name a subgroup of H, you can either list all of its elements, or if the subgroup is cyclic, you can name it using the notation $\langle a \rangle$.)

3. (14 points) It is a fact that $H = \{e, (123), (132)\}$ is a subgroup of S_4. (In other words, you may take it as given that H is a subgroup of S_4, and you do not have to prove it.)

(a) Write down all of the elements of the coset aH, for $a = (1243)$.

(b) Write down all of the elements of the coset Hb, for $b = (14)(23)$.

Show all your work.

For questions 4–6, you are given a statement. If the statement is true, you need only write "True", though a justification may earn you partial credit if the correct answer is "False". If the statement is false, write "False", and justify your answer **as specifically as possible**. (Do not just write "T" or "F", as you may not receive any credit; write out the entire word "True" or "False".)

4. (12 points) Let α be an element of S_7. If $|\alpha|$ is the order of α, then it is possible that $|\alpha| > 7$.

5. (12 points) Let $G = \langle a \rangle$ be a cyclic group of order 15. Then there are exactly 6 elements of G that generate G.

6. (12 points) If G and H are both groups of order 6, then it must be the case that G is isomorphic to H.

7. (14 points) **PROOF QUESTION.** Let α and β be elements of S_n. Use the definitions of even and odd permutations to prove that $\alpha\beta\alpha^{-1}\beta^{-1}$ is an even permutation.

8. (16 points) **PROOF QUESTION.** Let G be a group of order 27, and let H be a *proper* subgroup of G (i.e., $H \leq G$ and $H \neq G$). Prove that either H is cyclic or every nontrivial element of H has order 3.