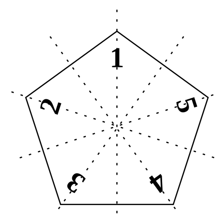
Sample Exam 1
Math 128A, Fall 2020

Note that this exam began with a "recite the definition" problem, which is a type of problem that your exam will not have.

1. (10 points) Let G be a group, and let a be an element of G. Define $\langle a \rangle$, the cyclic subgroup generated by a.

2. (12 points) Recall that in the group D_5, R_d is a counterclockwise rotation of d degrees, and for $1 \leq i \leq 5$, F_i is the reflection across the line shown below.



Compute $F_1 R_{144}$. Show all your work; in particular, show the results of any intermediate steps.

For questions 3–5, you are given a statement. If the statement is true, you need only write "True", though a justification may earn you partial credit if the correct answer is "False". If the statement is false, write "False", and justify your answer **as specifically as possible**. (Do not just write "T" or "F", as you may not receive any credit; write out the entire word "True" or "False".)

3. (12 points) Let G be a group, and let a and b be elements of G. It must be the case that $ab = ba$.

4. (12 points) Let H be a nonempty subset of $\mathbf{Z}$ that is closed under addition. Then it is possible that H is *not* a subgroup of G.

5. (12 points) Let G be a group, and let a be an element of G such that $a^{12} = e$. Then it must be the case that the order of a is 12.

6. (13 points) **PROOF QUESTION.** Let a and b be positive integers, and suppose k is an integer such that $\gcd(a, b)$ divides k. Prove that there exist $x, y \in \mathbf{Z}$ such that $ax + by = k$.

7. (13 points) **PROOF QUESTION.** Let G be a group, and suppose $a, t \in G$ are such that $t^{-1}at = a^{-1}$.

(a) Prove that $t^{-1}a^{-1}t = a$.

(b) Prove that $t^{-2}at^2 = a$.

Note: For part (b), you may use the result of part (a) even if you are not able to prove part (a).

8. (16 points) **PROOF QUESTION.** Let H be the subset of all $A \in GL(2, \mathbf{R})$ such that $\det A$ is a nonzero rational number. In other words, let

$$H = \{A \in GL(2, \mathbf{R}) \mid \det A \in \mathbf{Q}^*\} .$$

Prove that H is a subgroup of $GL(2, \mathbf{R})$. (Note: You may freely use what you know about rational numbers from previous math classes, including K–12 classes.)