Format and topics
Exam 3, Math 128A

General information. Exam 3 will be a timed test of 75 minutes, covering Chapters 7–11 of the text. (Note that Chs. 7 and 8 will be covered on Exam 3, even though they were also covered on Exam 2.) No books, notes, calculators, etc., are allowed. Most of the exam will rely on understanding the problem sets and the definitions and theorems that lie behind them. If you can do all of the homework, and you know and understand all of the definitions and the statements of all of the theorems we've studied, you should be in good shape.

You should not spend time memorizing proofs of theorems from the book, though understanding those proofs does help you understand the theorems. On the other hand, you should defintely spend time memorizing the *statements* of the important theorems in the text.

The exam will contain the same four types of questions as the previous one. (Remember to be as specific as possible on the true/false questions.) The exam will not be cumulative, per se, as there will be no questions that only concern material before Ch. 7. However, it will be assumed that you still understand the previous material; for example, it will be assumed that you know what groups and subgroups are, what $U(25)$ is, what an isomorphism is, and so on.

Definitions. The most important definitions we have covered are:

Ch. 7	aH, Ha, aHa^{-1}	left coset of H in G containing a
	right coset of H in G containing a	coset representative
	index of H in G	$\lvert G : H\rvert$
	$\lvert aH\rvert$, $\lvert Ha\rvert$	$\mathrm{stab}_G(i)$, stabilizer of i in G
	$\mathrm{orb}_G(i)$, orbit of i in G	
Ch. 8	external direct product	$G_1 \oplus \cdots \oplus G_n$
	$U_k(n)$	
Ch. 9	normal subgroup	$N \lhd G$
	factor group of G by H	quotient group
	G/H	internal direct product
Ch. 10	group homomorphism	kernel
	$\ker \varphi$	inverse image, pullback
	$\varphi^{-1}(x)$, $\varphi^{-1}(K)$	natural map
	natural homomorphism	
Ch. 11	determining the isomorphism class of G	partition

Examples. You will also need to be familiar with the key properties of the main examples we have discussed. The most important examples we have seen are:

Ch. 7 Computations of left and right cosets (pp. 140–141, PS06). Cosets of $SL(2, \mathbf{R})$ in $GL(2, \mathbf{R})$. A_4 has no subgroup of order 6. Group of a cube, soccer ball, icosahedron.

Ch. 8 Classification of groups of order 4. Counting numbers of elements of a given order. Computation of $U(n)$ as a direct product of cyclic groups.

Ch. 9 Normal subgroups: subgroups of abelian groups, $Z(G)$, A_n in S_n, rotations in D_n, $SL(2, \mathbf{R})$ in $GL(2, \mathbf{R})$, etc. Factor groups: Z/nZ, $D_4/Z(D_4)$ (pp. 180–181), etc. Decompositions of $U(n)$ (p. 190).

Ch. 10 Homomorphisms: isomorphisms, determinant, $|x|$, derivative, $Z \to Z_n$, wrapping function, sign of a permutation, etc.

Ch. 11 Abelian groups of order p^n ($1 \le n \le 4$), order $1176 = 2^3 \cdot 3 \cdot 7^2$. Determining the isomorphism class of subgroups of $U(n)$ (pp. 220–221).

Theorems, results, algorithms. The most important theorems, results, and algorithms we have covered are listed below. You should understand all of these results, and you should be able to state any theorem clearly and precisely. You don't have to memorize theorems by number or page number; however, you should be able to state a theorem, given a reasonable identification of the theorem (either a name or a vague description).

Ch. 7 Properties of cosets. Lagrange's Theorem; $|G : H| = |G| \,/\, |H|$, $|a|$ divides $|G|$, prime order groups are cyclic, $a^{|G|} = e$, Fermat's Little Theorem. Classification of groups of order $2p$; groups of order $n \leq 7$. Orbit-Stabilizer Theorem.

Ch. 8 Order of an element of a direct product. Criterion for $Z_m \oplus Z_n$ to be cyclic. $U(n)$ as an external direct product.

Ch. 9 Normal subgroup test. G/H is a group for H normal. The G/Z Theorem; $G/Z(G) \approx \text{Inn}(G)$; Cauchy's Theorem for Abelian Groups. Internal direct product is isomorphic to external direct product.

Ch. 10 Basic properties of homomorphisms (Thms. 10.1 and 10.2). Kernels are normal. First Isomorphism Theorem; consequences (Corollary). Normal subgroups are kernels.

Ch. 11 Fundamental Theorem of Finite Abelian Groups. Existence of Subgroups of Abelian Groups.

Not on exam. (Ch. 8) Applications (cryptography, genetics, electric circuits). Gauss' classification of $U(p^n)$ for p prime (bottom of p. 158). (Ch. 9) Group representations. (Ch. 11) "Greedy algorithm" for isomorphism class. Proof of the fundamental theorem (Lemmas 1–4).

Good luck.