Format and topics
Final exam, Math 128A

General information. The final will be somewhat less than twice as long as our in-class exams, with 120 minutes in which to complete it. The final will be **cumulative**; in other words, the final will cover the topics on this sheet and the topics on the previous three review sheets. However, the exam will somewhat emphasize the material listed here from Chapters 11, 12, and 14 and PS10–11.

As always, most of the exam will rely on understanding the problem sets and the definitions and theorems that lie behind them. If you can do all of the homework, and you know and understand all of the definitions and the statements of all of the theorems we've studied, you should be in good shape. You should not spend time memorizing proofs of theorems from the book, though understanding those proofs may help you understand the theorems. On the other hand, you should defintely spend time memorizing the *statements* of the important theorems in the text.

As usual: You will be allowed to use one page of notes, but no other aids (calculators, etc.), and there will be four basic types of questions, namely, computations, statements of definitions and theorems, proofs, and true/false with justification.

Definitions. The most important definitions we have covered are:

Ch. 11	determining the isomorphism class of G	partition
Ch. 12	ring	commutative ring
	unity, 1	unit, a^{-1}
	divide, factor, $a \mid b$	$R[x]$ (R a comm. ring)
	$M_n(R)$ (R a comm. ring)	direct sum $R_1 \oplus \cdots \oplus R_n$
	subring	Gaussian integers $\mathbf{Z}[i]$
	$n\mathbf{Z}$	
Ch. 14	ideal	proper ideal
	$\langle a_1, \ldots, a_n \rangle$	ideal generated by $a_1, \ldots, a_n$
	factor ring R/A	

Examples. You will also need to be familiar with the key properties of the main examples we have discussed. The most important examples we have seen are:

Ch. 11 Abelian groups of order p^n ($1 \leq n \leq 4$), order $1176 = 2^3 \cdot 3 \cdot 7^2$. Determining the isomorphism class of subgroups of $U(n)$ (pp. 215–216).

Ch. 12 $\mathbf{Z}$, $\mathbf{Q}$, $\mathbf{C}$, $\mathbf{R}$, $\mathbf{Z}_n$, $R[x]$, $M_n(R)$, $n\mathbf{Z}$, real-valued functions, $\mathbf{Z}[i]$, $R_1 \oplus \cdots \oplus R_n$. Trivial subring, diagonal matrices, upper triangular matrices.

Ch. 14 Ideals: R and $\{0\}$ in R, $n\mathbf{Z}$ in $\mathbf{Z}$, $\langle a_1, \ldots, a_n \rangle$ in R (examples of this class). Non-ideals: even-degree polynomials in $R[x]$. Factor rings: $\mathbf{Z}/n\mathbf{Z}$, $\mathbf{Z}[i]/\langle 2-i \rangle$, $\mathbf{R}[x]/\langle x^2+1 \rangle$.

Theorems, results, algorithms. The most important theorems, results, and algorithms we have covered are listed below. You should understand all of these results, and you should be able to state any theorem clearly and precisely. You don't have to memorize theorems by number or page number; however, you should be able to state a theorem, given a reasonable identification of the theorem (either a name or a vague description).

Ch. 11 Fundamental Theorem of Finite Abelian Groups. Existence of Subgroups of Abelian Groups.

Ch. 12 Rules of multiplication. Unity and inverses unique. Subring Test.

Ch. 14 Ideal test. Factor rings well-defined.

Not on exam. (Ch. 11) "Greedy algorithm" for isomorphism class. Proof of the fundamental theorem. (Ch. 14) Material on prime/maximal ideals vs. quotients that are integral domains/fields.

Good luck.