Math 128a, problem set 11
Outline due: Wed Dec 01
Due: Mon Dec 06
Last revision due: TBA

Problems to be done, but not turned in: (Ch. 13) 3, 9, 13, 25, 35, 41, 45, 47, 57; (Ch. 14) 5, 9, 15, 25, 29, 37, 41, 51, 55, 63.

Fun: (Ch. 13) 60.

Extra definition: For any set X, define $\mathbf{F}(X)$ to be the ring of all real-valued functions $f : X \to \mathbf{R}$, with addition and multiplication defined pointwise (i.e., $(f+g)(x) = f(x)+g(x)$ and $(fg)(x) = f(x)g(x)$). It can be shown that $\mathbf{F}(X)$ is a ring; you may take this as given.

Problems to be turned in:

1. Consider the ring $\mathbf{F}(\mathbf{R})$ defined above (taking $X = \mathbf{R}$). You may want to consult a calculus textbook for this problem.
   (a) Define $C(\mathbf{R})$ to be the set of all continuous functions $f : \mathbf{R} \to \mathbf{R}$. State the two theorems from calculus that must be proven to show that $C(\mathbf{R})$ is a subring of $\mathbf{F}(\mathbf{R})$.
   (b) Define $D(\mathbf{R})$ to be the set of all differentiable functions $f : \mathbf{R} \to \mathbf{R}$ (i.e., functions where $f'(x)$ exists for all $x \in \mathbf{R}$). State the two theorems from calculus that must be proven to show that $D(\mathbf{R})$ is a subring of $\mathbf{F}(\mathbf{R})$.
2. Find all units and zero-divisors in the ring $\mathbf{Z}_4 \oplus \mathbf{Z}_6$.
3. Let D be an integral domain with unity 1, and let S be the set of all elements of D that are their own inverses under multiplication. List all elements of S, and prove your answer.
4. Consider $\mathbf{F}(\mathbf{R})$, as defined above, and its subrings $C(\mathbf{R})$ and $D(\mathbf{R})$ from problem 1.
   (a) Prove that $\mathbf{F}(\mathbf{R})$ is not an integral domain.
   (b) Is $C(\mathbf{R})$ an integral domain? If so, state the theorem from calculus that proves that it is; if not, draw or describe one zero-divisor in $C(\mathbf{R})$.
   (c) (Extra credit) Is $D(\mathbf{R})$ an integral domain? Prove or disprove.
5. (Ch. 13) 40.
6. Find a subring of $\mathbf{Z} \oplus \mathbf{R}$ that is not an ideal of $\mathbf{Z} \oplus \mathbf{R}$. (Prove your answer.)
7. (Ch. 14) 12.