

Math 128A, problem set 10
Outline due: Wed Nov 24
Due: Mon Nov 29
Last revision due: TBA

Problems to be done, but not turned in: (Ch. 11) 3, 7, 11, 15, 21, 27, 29; (Ch. 12) 1, 3, 9, 13, 15, 17, 19, 25, 41, 49.

Fun: (Ch. 12) 33.

Problems to be turned in:

1. Let s and t be positive integers such that $\gcd(s, t) = 1$.

(a) Define a function $\varphi : U(st) \rightarrow U(s) \oplus U(t)$ by the formula

$$\varphi(x \pmod{st}) = (x \pmod{s}, x \pmod{t}).$$

(The PTBDBNTI from Ch. 0 show that φ is well-defined; you do not need to check this.)

Prove that φ is a homomorphism, and find $\ker \varphi$, with proof. (Suggestion: Use the PTBDBNTI from Ch. 0.)

(b) Use the First Isomorphism Theorem to prove that $U(st) \approx U(s) \oplus U(t)$. (Suggestion: Use PS01.)

2. Find all abelian groups (up to isomorphism) of order 720.

3. (Ch. 11) 12.

4. (Ch. 12) 6.

5. (Ch. 12) 24.

6. (Ch. 12) 40.

7. Let R be a ring such that $x^2 = x$ for all $x \in R$.

(a) Prove that $2a = a + a = 0$ for all $a \in R$.

(b) Prove that R is commutative.

(Suggestion: Consider $(a + b)^2$, etc.)