

Math 128A, problem set 08
CORRECTED THU OCT 28
Outline due: Wed Nov 03
Due: Mon Nov 08
Last revision due: Wed Dec 08

Problems to be done, but not turned in: (Ch. 8) 27, 37, 47, 59; (Ch. 9) 3, 7, 9, 15, 21, 25, 37, 41, 53, 61, 65, 71.

Fun: (Ch. 8) 33; (Ch. 9) 36, 62.

Problems to be turned in:

1. (Ch. 8) 44. Prove your assertion.
2. (Ch. 8) 56.
3. (Ch. 9) 12.
4. Let $G = \mathbf{Z}_8 \oplus \mathbf{Z}_2$ and let $H = \langle (2, 1) \rangle$.
 - (a) What is the order of the element $(2, 0) + H$ in the factor group G/H ?
 - (b) Is G/H cyclic? Prove your answer.
5. (Ch. 9) 26.
6. Let N be a normal subgroup of G , and suppose that $a \in G$. If aN has order 4 in the group G/N , and $|N| = 12$, what are the possibilities for the order of a ?
7. (Ch. 9) 50. (You may take it as given that H is a subgroup of G , and that H is normal in G , since G is abelian.)
8. Let n be an integer greater than 2. Recall (PS02, class) that for $r = R_{360/n}$ and f any reflection in D_n , we know that $r^n = f^2 = e$ and $frf^{-1} = r^{-1}$, and that

$$D_n = \{r^i, r^i f \mid 0 \leq i < n\}.$$

- (a) Prove that $\langle f \rangle$ is *not* normal in D_n .
- (b) Let k be a divisor of n , and let $H = \langle r^k \rangle$. Prove that H is normal in D_n .