Math 128A, problem set 08
Outline due: Mon Nov 02
Due: Wed Nov 04
Last revision due: Mon Dec 14

Problems to be done, but not turned in: (Ch. 8) 19–81 odd. (Ch. 9) 1–71 odd.
Fun: (Ch. 8) 35, 60; (Ch. 9) 36, 64.

Problems to be turned in:

1. Is $\mathbf{Z}_{15} \oplus \mathbf{Z}_{21} \oplus \mathbf{Z}_{63}$ isomorphic to $\mathbf{Z}_{315} \oplus \mathbf{Z}_7 \oplus \mathbf{Z}_9$? Is $\mathbf{Z}_{10} \oplus \mathbf{Z}_{14} \oplus \mathbf{Z}_{20}$ isomorphic to $\mathbf{Z}_2 \oplus \mathbf{Z}_{140} \oplus \mathbf{Z}_{10}$? Prove your assertions.
2. (Ch. 8) 72.
3. Recall that for $r = R_{30}$ and f any reflection in D_{12}, we have that $r^{12} = f^2 = e$ and $frf^{-1} = r^{-1}$.
   (a) Prove that $D_{12} = \left\{r^i, r^i f \mid 0 \leq i < 12\right\}$, or in other words, every element of D_{12} can be expressed uniquely in the form $r^i f^j$, where $0 \leq i < 12$ and $j = 0$ or 1. (Suggestion: What are the right cosets of $\langle r \rangle$?)
   (b) Prove that $\langle f \rangle$ is *not* normal in D_{12}.
   (c) Let k be a divisor of 12, and let $H = \left\langle r^k \right\rangle$. Use the previous parts of this problem and the relations $r^{12} = f^2 = e$ and $frf^{-1} = r^{-1}$ to prove that H is normal in D_{12}.
4. (Ch. 9) 12.
5. Let $G = \mathbf{Z}_{125} \oplus \mathbf{Z}_5$ and let $H = \langle (5, 1) \rangle$.
   (a) What is the order of the element $(5, 0) + H$ in the factor group G/H? Explain.
   (b) Is G/H cyclic? Prove your answer.
6. Let N be a normal subgroup of G, and suppose that $a \in G$. If aN has order 6 in the group G/N, and $|N| = 18$, what are the possibilities for the order of a?
7. (Ch. 9) 54.
8. **THIS PROBLEM CUT FROM PROBLEM SET** (Ch. 9) 62(b,c). (You can take part (a) as given.)