Math 128a, problem set 06
Outline due: Wed Oct 13
Due: Mon Oct 18
Last revision due: Mon Nov 15

Problems to be done, but not turned in: (Ch. 6) 7, 15, 17, 23, 27, 33, 35; (Ch. 7) 1, 3, 5, 9, 15.

Fun: (Ch. 6) 44.

Problems to be turned in:

1. Does there exist an automorphism $\varphi : \mathbf{Z}_{71} \to \mathbf{Z}_{71}$ such that $\varphi(13) = 25$? If so, describe *all* such φ as precisely as possible, with proof; if not, prove that no such φ exists.

2. Consider the groups $U(16)$, $U(20)$, and $U(24)$. For any two of them that you think are *not* isomorphic, prove that they are not isomorphic.

3. Find three groups G, H, K of order 120 such that $G \not\approx H$, $H \not\approx K$, and $G \not\approx K$. Prove your result.

4. Consider the group D_6, using our standard notation.

   (a) Let $K = \{e, F_{12}\} = \langle F_{12} \rangle$. List all of the left cosets of K and all of the right cosets of K.

   (b) Let $H = \{e, R_{120}, R_{240}\} = \langle R_{120} \rangle$. List all of the left cosets of H and all of the right cosets of H. Do you see any significant qualitative differences between this example and the previous one? Explain.

5. (Ch. 7) 6.

6. Let G be a group, and let H and K be subgroups of G such that $|H| = 60$ and $|K| = 76$. What are the possibilities for the order of $H \cap K$? Generalize.

7. (a) Let G be a group such that every nontrivial element of G has order 2. Prove that G is abelian.

   (b) Now let G be a group of order 8. Prove that if G is *not* abelian, then G must have an element of order 4.