

Math 128A, problem set 05
Outline due: Wed Oct 06
Due: Mon Oct 11
Last revision due: Mon Nov 15

Problems to be done, but not turned in: (Ch. 5) 9, 11, 13, 19, 27, 31, 41, 43; (Ch. 6) 1, 3, 5.

Fun: (Ch. 5) 54.

Problems to be turned in:

1. The Mind-Switcher (tm) is a machine that, when applied to bodies X and Y, switches the mind of X and the mind of Y. Unfortunately, the Mind-Switcher has the inconvenient flaw that it can only be applied to a given pair of bodies once.
 - (a) One day, Ami and Blender are playing around and switch their minds. To their horror, they discover that they can't just switch back. Fortunately, just then, Ceela and Droidberg come along to save the day. How? (Remember, any pair of bodies can only be switched once.)
 - (b) The 17 Clones of Fried then start playing with the Mind-Switcher. First clones 1 and 2 use the machine, then 2 and 3, and so on, up to 16 and 17. What is the final effect of this sequence of switches?
 - (c) Of course, the 17 clones now want to switch back to their original position. Luckily, Geese and High-Rise of the Glob-Trotters show up in the nick of time. How can Geese and High-Rise restore everyone back to their original body? (Suggestion: Try something like $(G\ 1)$, then $(G\ 2)$, etc.)

For all parts, show all your work, in cycle notation.

2. (Ch. 5) 34.
3. Choose two elements $\alpha, \beta \in S_6$ with different cycle shapes, and compare the cycle shapes of $\alpha\beta$ and $\beta\alpha$. Do this for at least 4 such pairs. What pattern do you see?
4. (Ch. 5) 22.
5.
 - (a) Let G be a group, and let a be an element of G . Prove that the map $f : G \rightarrow G$ defined by $f(x) = ax$ is a bijection.
 - (b) Let H be a subgroup of S_n that contains at least one odd permutation. Prove that the order of H is even.
6. (Ch. 6) 6.
7.
 - (a) Prove that the map $\varphi_3 : U(11) \rightarrow U(11)$ defined by $\varphi_3(x) = x^3$ is an automorphism of $U(11)$.
 - (b) For which n is the map $\varphi_n : U(11) \rightarrow U(11)$ defined by $\varphi_n(x) = x^n$ an automorphism of $U(11)$? Find a pattern. (You do not need to prove your answer to 7b, but show some work or evidence.)