Math 128A, problem set 04
CORRECTED WED SEP 28
Outline due: Wed Sep 29
Due: Mon Oct 04
Last revision due: Mon Oct 18

Problems to be done, but not turned in: (Ch. 4) 15, 19, 23, 33, 39, 45; (Ch. 5) 1, 3, 7, 17.

Fun: (Ch. 4) 42, 56, 66.

Problems to be turned in:

1. Justify all answers.
   (a) (Ch. 4) 10.
   (b) (Ch. 4) 12.

2. Find the subgroup lattices for $\mathbf{Z}_5$, $\mathbf{Z}_{10}$, $\mathbf{Z}_{70}$, and $\mathbf{Z}_{210}$. Generalize as much as you can.

3. (Ch. 4) 48. You do not have to prove your generalization.

4. (Ch. 4) 60. Prove your answer.

5. (a) Let G be an abelian group of order 55 such that $x^{55} = e$ for all $x \in G$. Prove that G is cyclic.
   (b) Let G be an abelian group of order 9 such that $x^9 = e$ for all $x \in G$, and suppose that G is *not* cyclic. What can you say about G?

6. Let $\alpha = (1\ 3\ 4\ 5)(2\ 7\ 8)(9\ 10)$ and $\beta = (1\ 7\ 3\ 10\ 5)(2\ 8\ 6\ 9)$ be elements of S_{10}.
   (a) Compute $\alpha\beta$, in cycle form.
   (b) Find the orders of α, β, and $\alpha\beta$.

7. The *cycle shape* of $\alpha \in S_n$ is the set (or actually, multiset) of the lengths of the cycles obtained when α is expressed as a product of disjoint cycles (see pp. 102–103).
   (a) Find all possible cycle shapes of elements of S_8, and find the orders of the elements with those cycle shapes.
   (b) Find all possible cycle shapes of elements of A_8.