Math 128A, problem set 03
Outline due: Wed Sep 15
Due: Mon Sep 20
Last revision due: Mon Oct 18

Problems to be done, but not turned in: (Ch. 3) 23, 25, 29, 33, 37, 41, 47, 53; (Ch. 4) 3, 5, 9, 13.

Fun: (Ch. 3) 44, 60.

Problems to be turned in:

1. (a) Consider $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$, and $C = \begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix}$. Find the orders of A, B, and C.

   (b) If G is a group, $x, y \in G$, and both x and y have finite order, is it always the case that xy has finite order? Prove or give a counterexample.

2. (Ch. 3) 40.

3. List all cyclic subgroups of D_6.

4. Let K be a subgroup of $\mathbf{R}^*$ (the multiplicative group of the nonzero reals), and let

$$H = \{A \in GL(3, \mathbf{R}) \mid \det A \in K\}.$$

   Prove that H is a subgroup of $GL(3, \mathbf{R})$.

5. Let G be a group, and let both H and K be subgroups of G.

   (a) Is it always the case that $H \cup K$ is a subgroup of G? Prove or give a counterexample.

   (b) Is it always the case that $H \cap K$ is a subgroup of G? Prove or give a counterexample.

6. Let G be a group, and let both H and K be subgroups of G. Define

$$HK = \{hk \mid h \in H, k \in K\}.$$

   (a) Give an example to show that HK may not be a subgroup of G.

   (b) Now suppose further that $hk = kh$ for all $h \in H$ and $k \in K$. Prove that HK is a subgroup of G.

7. (a) (Ch. 4) 4.

   (b) (Ch. 4) 6.