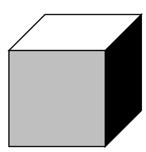
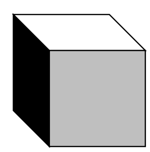
Math 128A, problem set 01
Outline due: Mon Aug 30
Due: Wed Sep 01
Last revision due: Mon Sep 20

Problems to be done, but not turned in: (Ch. 0) 1, 3, 11, 13, 17, 19, 41, 49; (Ch. 1) 1, 7, 9, 11, 19.

Fun: (Ch. 0) 14, 56; (Ch. 1) 16.

Problems to be turned in:

1. (Ch. 0) 52.
2. Let t, a, and b be positive integers.
   (a) Prove that if t divides a and t divides b, then t divides $a+b$.
   (b) Without using the Fundamental Theorem of Arithmetic, prove that if t divides a and t divides b, then t divides $\gcd(a,b)$.
3. Prove that if a and b are integers, s and t are positive integers, and $\gcd(s,t)=1$, then there exists some integer n such that $n=a \pmod{s}$ and $n=b \pmod{t}$. Suggestion: Note that $n=a \pmod{s}$ exactly when $n=a+sx$ for some integer x.
4. (Ch. 0) 40. Explain (prove) your answer.
5. (a) (Ch. 1) 6. (Draw a picture to show which rotation this would be.)
   (b) Choose two distinct reflections f and g in D_5, and compute fg in your notation.
   (c) (Ch. 1) 8. (Draw a picture to show which reflection this would be.)
   (d) Choose a non-trivial rotation r and a reflection f in D_5, and compute rf.
6. (Ch. 1) 22.
7. Consider a cube with its top and bottom faces painted white, its front and back faces painted gray, and its left and right faces painted black, as shown below.

   (front right view)  (front left view) 

   Note that, for example, a 180-degree rotation around the axis going through the centers of the top and bottom faces is a (three-dimensional) rotation symmetry of the cube that preserves the coloring of the faces.
   (a) Draw or otherwise describe one rotation symmetry of the unpainted cube that does *not* preserve the above coloring of its faces.
   (b) List all of the rotation symmetries of the cube that preserve the above coloring of its faces. (In particular, give each symmetry a name.)
   (c) You may take it as given that the rotation symmetries of the cube that preserve the coloring form a group G. Using your notation from 7b, construct the Cayley table of G.