Math 128A, Wed Sep 02

Mon = Labor Day

- Use a laptop or desktop with a large screen so you can read these words clearly.
- In general, please turn off your camera and mute yourself.
- Exception: When we do groupwork, please turn both your camera and mic on. (Groupwork will not be recorded.)
- Please always have the chat window open to ask questions.
- Reading for ~~Mon~~ Wed: Ch. 4.
- Outline for PS02 due 11pm, completed version due Wed Sep 09.
- Outline for PS03 due Wed Sep 09.
- Next problem session Fri Sep 04, 10:00–noon on Zoom.

Subgroups, cont.

G a group.

Theorem (Enhanced Two-Step Subgroup Test)

Suppose $H \subseteq G$. TFAE:

- *H is a sub**group** of G.*
- *The following all hold:*
  0. *H is nonempty, i.e., there exists some element of G in H;*
  1. *H is closed under operation, i.e., if $a, b \in H$, then $ab \in H$; and*
  2. *H is closed under taking inverses, i.e., if $a \in H$, then $a^{-1} \in H$.*

If-then!

Can combine steps 1 and 2 to make the One-Step Subgroup Test:

- If $a, b \in H$, then $ab^{-1} \in H$. $\Leftarrow$ 1. + 2.

$a = e \Rightarrow 2.$

$b = x^{-1} \Rightarrow 1.$ (w/ a, x)

Method for proving that $H \subseteq G$ is a sub**group** of G

set-builder notation to define a set

$\{$ what elements look like $|$ what conditions elements must satisfy $\}$

Suppose H has a definition of the form $\{\text{foo} \mid \text{bar}\}$. To apply the Two-Step Subgroup Test:

- Write out steps 0, 1, 2 as if-then statements and set up **A**ssumptions and **C**onclusions.
- Rewrite **A** and **C** using $\{\text{foo} \mid \text{bar}\}$ definition of H.
- Fill in the middle.

The cyclic subgroup generated by $a \in G$

Theorem

G a group, $a \in G$. Then

Elts of H have form a^n

$$H = \langle a \rangle = \{a^n \mid n \in \mathbf{Z}\}$$

Where n is some integer

is a subgroup of G.

$n = 1$ is an integer

So $a^1 \in H$

① There is something in H

Assume: $a^c, a^d \in H \mid c, d \in \mathbb{Z}$

$$(a^c)(a^d) = a^{c+d}$$

B/c $c+d \in \mathbb{Z}$

Conclusion: $a^c a^d \in H$

2: A: $x \in H$

So $x = a^m$

$x^{-1} = a^{-m}$

b/c $-m \in \mathbb{Z}$

C: $x^{-1} \in H$

The centralizer of $a \in G$

Theorem

G a group, $a \in G$. Then

$$C(a) = \{g \in G \mid ga = ag\}$$

is a subgroup of G.