Definition of a group

A **group** is a set G with a **binary operation** (written here as multiplication) such that:

(Associativity) For all $a, b, c \in G$, $(ab)c = a(bc)$.

(Identity) There exists $e \in G$ such that $ae = ea = a$ for all $a \in G$.

(Inverses) For all $a \in G$, there exists $b \in G$ such that $ab = ba = e$.

Definition of a ring

A **ring** is a set R with binary operations $+$ and $\cdot$ (multiplication) such that:

(Abelian group, 4 axioms) The operation $+$ gives R the structure of an abelian group, with (additive) identity 0 and the inverse of a written $-a$.

(Associativity of multiplication) For all $a, b, c \in R$, $(ab)c = a(bc)$.

(Distributive) For all $a, b, c \in R$, $a(b + c) = ab + ac$ and $(a + b)c = ac + bc$.

Other types of rings include:

(Rings with unity) If there exists $1 \in R$ such that $1a = a1 = a$ for all $a \in R$ and $1 \neq 0$, we say that 1 is a **unity** (or **multiplicative identity**) in R.

(Commutative rings) If $ab = ba$ for all $a, b \in R$, we say that R is **commutative**.