Sample Exam 1
Math 127, Fall 2024

1. (12 points) Briefly explain how you can be sure that -2 is **not** a quadratic residue mod 7. Show all your work (if any).

2. (12 points) Let $f(x), g(x)$ be polynomials in $R[x]$ for some coefficient ring R.

(a) Give an example of a coefficient ring R and nonzero $f(x), g(x)$ such that $f(x)g(x) = 0$. No explanation necessary.

(b) Now suppose R has the Zero Factor Property, and $h(x) = f(x)g(x)$. What can you say about $\deg h(x)$? No explanation necessary.

3. (12 points)

(a) Find the smallest positive integer n such that $4^n = 1$ in $\mathbf{Z}/(13)$. Show all your work.

(b) Is 4 primitive mod 13? Briefly (1 or 2 sentences) **EXPLAIN** your answer in terms of the definition of primitive.

4. (12 points) Use the Signed Euclidean Algorithm to find $\gcd(213, 135)$. Show all your work. (If you don't know/remember how to use the Signed Euclidean Algorithm, you can use the unsigned Euclidean Algorithm for partial credit.)

5. (13 points) Use the Euclidean Algorithm to find $\gcd(x^7 + x^3 + x^2 + x, x^5 + x^2)$ in $\mathbf{F}_2[x]$. Show all your work.

6. (13 points) Use the Euclidean Algorithm to find the multiplicative inverse of 28 in $\mathbf{Z}/(103)$. Show all your work.

7. (13 points) Consider the following (silly) definition. (Assume all matrices have real number entries.)

> To **schmultiply** a matrix A by a column vector $\begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$, do the following: Multiply each entry of A by x_1; then multiply each entry of A by x_2; and so on, all the way through x_n.

(a) What is the result of schmultiplying the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ by the column vector $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$?

(b) Given a big-O estimate of the number of (real number) multiplications needed to schmultiply an $n \times n$ matrix A by a column vector $\begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$. Express your answer in the form $O(n^k)$ for some integer k, and **EXPLAIN** your answer in a few sentences.

8. (13 points) For $a \in \mathbf{Z}$, use the definition of "divides" (and not other results from the homework, etc.) to prove that if 7 divides a and 21 divides b, then 7 divides $5a + 9b$.