

**Sample Exam 1**  
**Math 127, Spring 2023**

1. (12 points) Let  $R$  be a ring in which  $0 \neq 1$ , and let  $a$  and  $b$  be elements of  $R$ .
- (a) Define what it means for  $b$  to be the (multiplicative) inverse of  $a$ .
  - (b) Define what it means for  $a$  to be a unit.
  - (c) Define what it means for  $R$  to be a field.

2. (12 points) Use the Signed Euclidean Algorithm to find  $\gcd(201, 111)$ . Show all your work. (If you don't know/remember how to use the Signed Euclidean Algorithm, you can use the unsigned Euclidean Algorithm for partial credit.)

3. (12 points)

- (a) Define what it means for  $a \in \mathbf{Z}/(47)$  to be a quadratic residue mod 47.
- (b) Suppose you want to list all quadratic residues mod 47. Briefly (1 or 2 sentences) **EXPLAIN** why you only have to compute 23 squares (mod 47), and not 46, to do this.

4. (12 points)

$n$	1	2	3	4	5	6	7	8	9	10
$5^n \pmod{11}$										

- (a) Fill in the above table, where all powers of 5 (i.e., all  $5^n$ ) are computed in  $\mathbf{Z}/(11)$ .
- (b) Is 5 primitive mod 11? Briefly (1 or 2 sentences) **EXPLAIN** your answer in terms of the definition of primitive.

5. (13 points) For  $a, d, k \in \mathbf{Z}$ , use the definition of “divides” (and not other results from the homework, etc.) to prove that if  $d$  divides  $a$ , then  $d$  divides  $a - dk$ .

6. (13 points) Use the Euclidean Algorithm to find the multiplicative inverse of 23 in  $\mathbf{Z}/(89)$ . Show all your work.

7. (13 points) Use the Euclidean Algorithm to find  $\gcd(x^6 + x^5 + x^3 + x^2 + x, x^5 + x^4)$  in  $\mathbf{F}_2[x]$ . Show all your work.

8. (13 points) For each  $n$ , consider the following procedure on an  $n \times n$  chessboard: Put 1 grain of rice on the first square, 2 grains of rice on the second square, 3 grains on the third square, 4 grains on the 4th square, and so on, for each of the  $n^2$  squares on the board.

- (a) For a  $3 \times 3$  chessboard, how many total grains of rice end up on the board? Express your answer as a sum or product, which you don't need to actually compute.
- (b) Given a big-O estimate of the total number of grains of rice that end up on an  $n \times n$  board. Express your answer in the form  $O(n^k)$  for some constant  $k$ .