Format and topics for final exam
Math 127

General information. The final will be 135 minutes long, i.e., a little less than twice as long as our in-class exams. The exam will be **cumulative**; in other words, the final will cover the topics on this sheet and also on the previous three review sheets. (Exception: Ch. 4 will not be covered on the final.) However, the exam will somewhat emphasize the material listed here from Sections 9.2–9.4, 10.1–10.3, and 11.2–11.3 (i.e., PS10–11).

As before, most of the exam will rely on understanding the problem sets and the definitions and theorems that lie behind them. If you can do all of the homework, and you know and understand all of the definitions and the statements of all of the theorems we've studied, you should be in good shape. You should not spend time memorizing proofs of theorems from the book, though understanding those proofs does help you understand the theorems. On the other hand, you should defintely spend time memorizing the *statements* of the important theorems in the text.

The final will follow the usual ground rules. In particular, no books, notes, or calculators are allowed, and the same types of questions may appear.

Definitions. The most important definitions and symbols we have covered are:

9.2	natural primitive Nth root of unity	ω_N, ω
9.3	signal	basic trigonometric signals
9.4	DFT	inverse DFT
9.5	convolution	
10.1	group	abelian group
	subgroup	C_n
	cyclic subgroup generated by a	$\langle a \rangle$
10.2	order of an element	
10.3	left multiplicative coset	coset representative
	partition	transversal
11.2	subgroup fill stage of FFT	
11.3	subgroup subdiagram	circuit diagram

Theorems, results, algorithms. The most important theorems, results, and algorithms we have covered are listed below. You should understand all of these results, and you should be able to state any theorem clearly and correctly. More precisely, you should be able to state any of the theorems, given a reasonable identification of the theorem (either its name or a vague description).

Sect. 9.2 Nth roots of unity are powers of ω_N (Thm. 9.2.3); $1 + \omega_N + \cdots + \omega_N^{N-1} = 0$.

Sect. 9.3 Orthogonality Lemma 9.3.4

Sect. 9.4 Inversion Theorem 9.4.4. Matrix notation for DFT (Rems. 9.4.2 and 9.4.5).

Sect. 9.5 Convolution is polynomial multiplication (Thm. 9.5.2). Substitution Lemma 9.5.3; DFT turns convolution into pointwise multiplication (Thm. 9.5.4).

Sect. 10.1 Subgroup Theorem 10.1.8. Subgroup properties of C_n (Thm. 10.1.10). Cyclic subgroup generated by a really is a subgroup (Thm. 10.1.12).

Sect. 10.2 If order of a is n: a^k depends only on $k \pmod n$ (Thm. 10.2.2); $a^k = 1$ iff n divides k (Cor. 10.2.3); $\langle a \rangle$ contains n elements (Cor. 10.2.4); order of a^k is $n/\gcd(k,n)$ (Thm. 10.2.5).

Sect. 10.3 If $b \in aH$, then $bH = aH$ (Thm. 10.3.3); either $aH = bH$ or $aH \cap bH = \emptyset$ (Cor. 10.3.5). Cosets partition G (Thm. 10.3.7). Order of subgroup divides order of group (Cor. 10.3.11); order of element divides order of group (Cor. 10.3.12).

Sect. 11.2 Generating C_d (Thm. 11.2.2). FFT in formula-based description (Alg. 11.2.4). Group-theoretic description of FFT (Rem. 11.2.9).

Sect. 11.3 Drawing subgroup subdiagrams and circuit diagrams (Alg. 11.3.4). FFT is $O(N \log N)$ (Thm. 11.3.5 and discussion beforehand).

Examples. You will also need to be familiar with the key properties of the main examples we have discussed. The most important examples we have seen are:

Sect. 9.3 Explicit expansions of $\hat{f}(k)$ in Prob. 9.4.1.
Sect. 10.1 Group of units of a ring R (Ex. 10.1.5). C_n as subgroup of $\mathbf{C}^\times$ (Thm. 10.1.10), as cyclic subgroup (Ex. 10.1.13).
Sect. 10.3 Cosets and representatives in $\mathbf{F}_{13}^\times$ (Exs. 10.3.2, 10.3.9).
Sect. 11.2 Generators for C_d (Exmp. 11.2.3). Degenerate example of FFT (Exmp. 11.2.5). FFTs for $N = 6$ (Exmp. 11.2.6), $N = 16$ (Exmp. 11.2.7).
Sect. 11.3 Circuit diagrams for $N = 6$ (Exmp. 11.3.1), $N = 16$ (Exmp. 11.3.2).

Not on exam. Ch. 4. You will be given the formulas that define the "subgroup fill" stage of the FFT, though it is probably better to understand the group theory-based description of Rem. 11.2.9.

Other. You should have a working familiarity with techniques and strategies for proof and logic tips, to the extent that we have done proofs in the homework.

Good luck.