Example: FFT for $N = 24$, $C_1 \le C_2 \le C_6 \le C_{12} \le C_{24}$

$n = 4$

Subgroups and transversals:

$\omega^{24} = 1$

$C_e = \omega^{N/e}$

$$\langle \omega^{24} \rangle \le \langle \omega^{12} \rangle \le \langle \omega^{4} \rangle \le \langle \omega^{2} \rangle \le \langle \omega \rangle$$

$$H_0 \qquad H_1 \qquad H_2 \qquad H_3 \qquad H_4$$

$\boxed{T}$			
$\{1, \omega^{12}\}$	$\{1, \omega^4, \omega^8\}$	$\{1, \omega^2\}$	$\{1, \omega\}$
$d = 2$	$d = 3$	$d = 2$	$d = 2$
$m = 24$	$m = 12$	$m = 4$	$m = 2$
$k = 12$	$k = 4$	$k = 2$	$k = 1$

$$H_2 = 1H_1 \cup \omega^4 H_1 \cup \omega^8 H_1$$

Conceptual description of subgroup fill, step i

In the subgroup fill part of step i, with "input subgroup" H_{i-1}, "output subgroup" H_i, and $H_{i-1} \leq H_i$:

- For the output corresponding to the jth element of H_i,
- We form a linear combination starting with the input corresponding to the jth element of H_{i-1},
- Offset by the elements of the transversal,
- With coefficients equal to the elements of the transversal raised to the $(-j)$th power.

Subgroup fill, Step 2:

$$H_1 = \{1 = \omega^0, \omega^{12}\} \qquad d = 3 \qquad H_2 = \{1, \omega^4, \omega^8, \omega^{12}, \omega^{16}, \omega^{20}\}$$

H_{i-1} $\qquad$ H_i

$$T = \{1, \omega^4, \omega^8\}$$

$$j=0 \quad y(0) = x(0) + x(4)(1) + x(8)(1)$$

$$j=1 \quad y(4) = x(12) + x(16)\,\omega^{-4} + x(20)\,\omega^{-8}$$

$$j=2 \quad y(8) = x(0) + x(4)\,\omega^{-8} + x(8)\,\omega^{-16}$$

$$j=3 \quad y(12) = x(12) + x(16)\,\omega^{-12} + x(20)(1)$$

$$j=4 \quad y(16) = x(8) + x(4)\,w^{-16} + x(8)\,w^{-8}$$

$$j=5 \quad y(20) = x(12) + x(16)\,w^{-20} + x(20)\,w^{-16}$$

Subgroup fill, Step 3:

$H_2 = \langle w^4 \rangle$ $\{1, w^2\}$ $H_3 = \langle w^2 \rangle$

jk jm $jm+k$

$j=0$ $y(0) = x(0) + x(2)$

$j=1$ $y(2) = x(4) + x(6)\, w^{-2}$

$y(4) = x(8) + x(10)\, w^{-4}$

$y(6) = x(12) + x(14)\, w^{-6}$

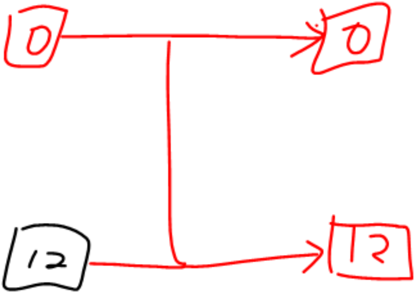
$y(8) = x(16) + x(18)\, w^{-8}$

$y(10) = x(20) + x(22)\, w^{-10}$

$$
\begin{aligned}
y(12) &= x(0) + x(2)\,\omega^{-12} \\
y(14) &= x(4) + x(6)\,\omega^{-14} \\
y(16) &= x(8) + x(10)\,\omega^{-16} \\
y(18) &= x(12) + x(14)\,\omega^{-18} \\
y(20) &= x(16) + x(18)\,\omega^{-20} \\
y(22) &= x(20) + x(22)\,\omega^{-22}
\end{aligned}
$$

Circuit diagrams: $N = 24$, $C_1 \leq C_2 \leq C_6 \leq C_{12} \leq C_{24}$

Step 1, subgroup subdiagram: $\langle w^{24} \rangle \leq \langle w^{12} \rangle$



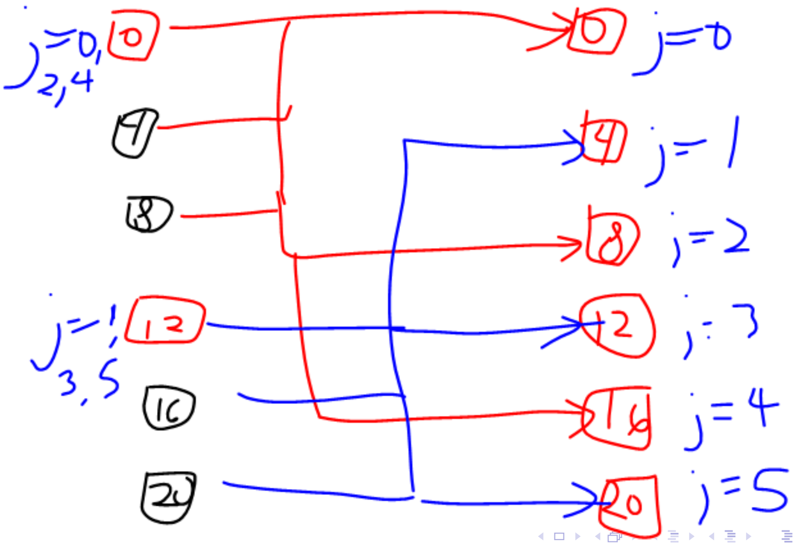
(+translates)

$$y(0) = x(0) + x(12)$$
$$y(12) = x(0) + x(12)\,w^{-12}$$

Step 2, subgroup subdiagram:

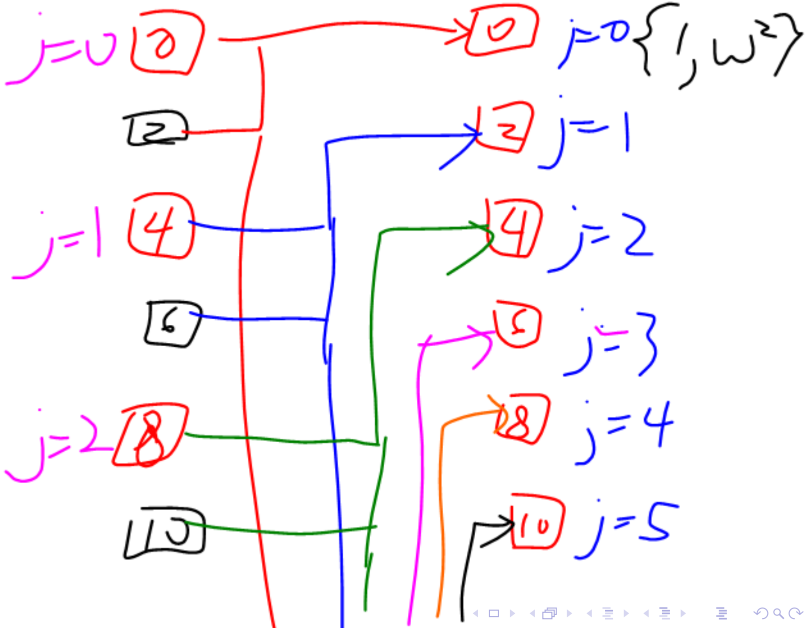
$T = \{1, w^4, w^8\}\langle w^{12}\rangle \leq \langle w^4\rangle$

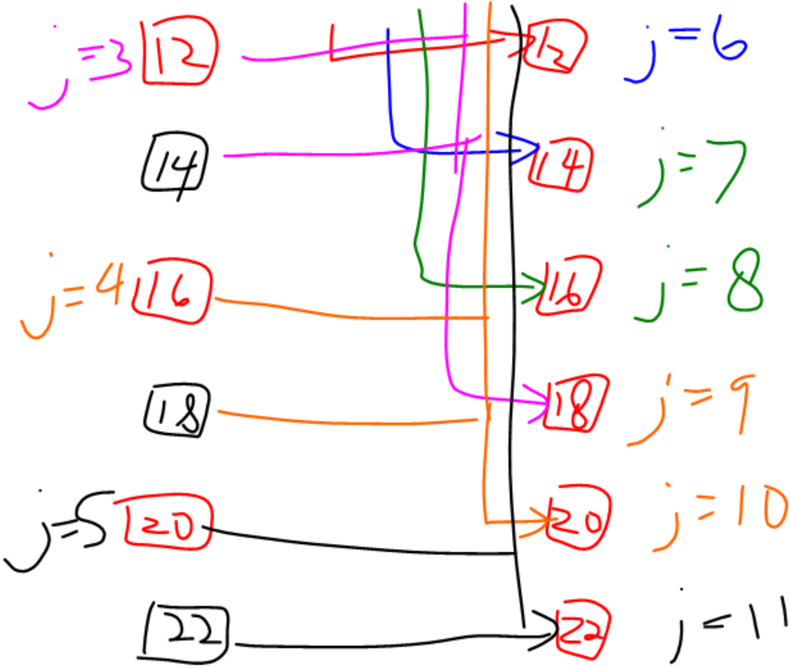
$d = 3$



Step 3, subgroup subdiagram:

$\langle \omega^4 \rangle \leq \langle \omega^2 \rangle$




j=3
12
14
j=4
16
18
j=5
20
22
12
j=6
14
j=7
16
j=8
18
j=9
20
j=10
22
j=11