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PS04 Outline

Part I: Definitions

- Defn: A permutation of a set A is a function from A to A that is both one-to-one and onto.
- Defn: A permutation group of a set A is a set of permutations of A that forms a group under function composition
- Defn: The set of all permutations of A is called the symmetric group of degree n and is denoted S_n
- Defn: Cycle notation is a commonly used notation to specify permutations.
- Defn: An expression of the form $(a_1, a_2, \ldots, a_m)$ is called a cycle of length m or an m-cycle.
- Defn: Various cycles that have no number in common can be expressed in disjoint cycle form.
- Defn: An n-cycle is denoted by $(\underline{n})$.
- Defn: A permutation that can be expressed as a product of an even number of 2-cycles is called an even permutation
- Defn: A permutation that can be expressed as a product of an odd number of 2-cycles is called an odd permutation.
- Defn: The group of even permutations of n symbols is denoted by A_n and is called the alternating group of degree n.
- Defn: A symmetric polynomial in the variables $x_1, x_2, \ldots, x_n$ is one that is unchanged under any transposition of two of the variables.
- Defn: An alternating polynomial is one that changes signs under any transposition of two of the variables.

Part II: Planning:

- Computation (PS04, 1a)

The final answer will be all generators of the subgroup $\langle 5 \rangle$ of $\mathbb{Z}$.

- Computation (PS04, 1b)

The final answer will be a list of all generators of $\langle a^5 \rangle$ of $\mathbb{Z}$.

· Computation (PS04, 1c)
The final answer will be the generators of subgroup H of $\mathbb{Z}_{60}$ where H has order 20.
· Computation (PS04, 1d)
The final answer will be the generators of subgroup H.
· Computation: (PS04, 2)
The final answer will be subgroup lattices for $\mathbb{Z}_5$, $\mathbb{Z}_{10}$, $\mathbb{Z}_{70}$, and $\mathbb{Z}_{770}$.
· Computation: (PS04, 3)
The final answer will be the possible orders of FF' for D_{21}.
· If-then (PS04, 4)
Assume: $|x|=n$
Conclude: $\langle x^r \rangle \subseteq \langle x^s \rangle$
· If-then (PS04, 5a)
Assume: $x^{49}=e$ for $x \in G$, G is an abelian group
Conclude: G is cyclic
· Computation: (PS 04, 5b)
The final answer will be more information about G.
· Computation: (PS04, 6a)
The final answer will be $\alpha\beta$ in cycle form
· Computation: (PS04, 6b)
The final answer will be the orders of α, β, and $\alpha\beta$.
· Computation: (PS 04, 7a)
The final answer will be all possible cycle shapes of elements of S_8 and their order.
· Computation: (PS04, 7b)
The final answer will be all possible cycle shapes of elements of A_8.