Math 127, Spring 2022 **Name:** ________________________

Exam 1

This test consists of 8 questions on 5 pages, totalling 100 points. You are not allowed to use books, notes, or calculators. Unless otherwise stated, you may take as given anything that has been proven in class, in the homework, or in the reading.

1. (12 points) Let R be a ring in which $0 \neq 1$, and let a and b be elements of R.

(a) Define what it means for b to be the (multiplicative) inverse of a.

(b) Define what it means for a to be a unit.

(c) Define what it means for R to be a field.

2. (12 points) Use the Signed Euclidean Algorithm to find $\gcd(201, 111)$. Show all your work. (If you don't know/remember how to use the Signed Euclidean Algorithm, you can use the unsigned Euclidean Algorithm for partial credit.)

3. (12 points)

(a) Define what it means for $a \in \mathbf{Z}/(47)$ to be a quadratic residue mod 47.

(b) Suppose you want to list all quadratic residues mod 47. Briefly (1 or 2 sentences) **EXPLAIN** why you only have to compute 23 squares (mod 47), and not 46, to do this.

4. (12 points)

n	1	2	3	4	5	6	7	8	9	10
5^n (mod 11)										

(a) Fill in the above table, where all powers of 5 (i.e., all 5^n) are computed in $\mathbf{Z}/(11)$.

(b) Is 5 primitive mod 11? Briefly (1 or 2 sentences) **EXPLAIN** your answer in terms of the definition of primitive.

5. (13 points) For $a, d, k \in \mathbf{Z}$, use the definition of "divides" (and not other results from the homework, etc.) to prove that if d divides a, then d divides $a - dk$.

6. (13 points) Use the Euclidean Algorithm to find the multiplicative inverse of 23 in $\mathbf{Z}/(89)$. Show all your work.

7. (13 points) Use the Euclidean Algorithm to find $\gcd(x^6 + x^5 + x^3 + x^2 + x, x^5 + x^4)$ in $\mathbf{F}_2[x]$. Show all your work.

8. (13 points) For each n, consider the following procedure on an $n \times n$ chessboard: Put 1 grain of rice on the first square, 2 grains of rice on the second square, 3 grains on the third square, 4 grains on the 4th square, and so on, for each of the n^2 squares on the board.

(a) For a 3×3 chessboard, how many total grains of rice end up on the board? Express your answer as a sum or product, which you don't need to actually compute.

(b) Given a big-O estimate of the total number of grains of rice that end up on an $n \times n$ board. Express your answer in the form $O(n^k)$ for some constant k.