Math 127, Spring 2021 **Exam 1**

This test consists of 7 questions on 1 pages, totalling 100 points. You are allowed to use one page of notes. Unless otherwise stated, you may take as given anything that has been proven in class, in the homework, or in the reading.

1. (13 points) Consider the polynomials $f(x) = x^5 + 2x^4 + 2x + 1$ and $g(x) = x^4 + x^3 + 2x^2 + 1$ in $\mathbf{F}_3[x]$. Find $\gcd(f(x), g(x))$ in $\mathbf{F}_3[x]$. Show all your work, making sure that at each stage of your calculation, you express each coefficient as 0, 1, 2, or -1.

2. (13 points) Use the Signed Euclidean Algorithm to find $\gcd(261, 204)$. Show all your work. (If you don't know/remember how to use the Signed Euclidean Algorithm, you can use the unsigned Euclidean Algorithm for partial credit.)

3. (13 points) Consider 5 as an element of $\mathbf{F}_{13}$.

(a) Find the powers 5^1, 5^2, etc., mod 13, until you see a pattern.

(b) Is 5 primitive mod 13? Briefly **EXPLAIN** your answer.

4. (13 points) Find the multiplicative inverse of 22 in $\mathbf{Z}/(103)$. Show all your work.

5. (16 points) Which elements of $\mathbf{Z}/(18)$ are units? Briefly **JUSTIFY** your answer by explaining the relevant results from the text.

6. (16 points) The goal of this problem is to estimate the complexity of multiplying two n-digit numbers, in terms of the number of single-digit multiplications. (For simplicity, we ignore additions.)

(a) List **EACH** single-digit multiplication required to multiply 123×456 using the standard grade-school algorithm (or any other algorithm you know). How **many** single-digit multiplications are there, in total?

(b) Generalize the above example to give a big-O estimate of the number of single-digit multiplications required to multiply two n-digit numbers.

7. (16 points) **PROOF QUESTION.** Suppose d and n are integers. Use the definition of divisibility (and not the results you proved on the homework) to prove that if d divides n, then d divides n^2.