Sample Exam 2
Math 126, Spring 2015

1. (8 points) State the Chinese Remainder Theorem.

2. (12 points) Find an integer n such that $0 \leq n < 47$ and $n \equiv 7^{4648} \pmod{47}$. Briefly **JUSTIFY** your answer.

3. (10 points) Let $m = 2^3 7^2 13$. Find the number of integers k such that $1 \leq k \leq m$ and $\gcd(k, m) = 1$. No explanation necessary, but show all your work. **DO NOT SIMPLIFY YOUR ANSWER.**

4. (10 points) Consider the congruence

$$10x \equiv 6 \pmod{28}.$$

If this congruence has at least one solution, find a largest possible set of incongruent solutions, showing all your work; if the congruence has no solutions, explain how you can be sure that it has no solutions.

For questions 5–8, you are given a statement. If the statement is true, you need only write "True", though a justification may earn you partial credit if the correct answer is "False". If the statement is false, write "False", and justify your answer **as specifically as possible**. (Do not just write "T" or "F", as you may not receive any credit; write out the entire word "True" or "False".)

5. (12 points) It is possible that there exist only finitely many primes whose last digit is 7.

6. (12 points) If n is an integer such that $n \geq 2$ and $2^n - 1$ is prime, then it must be the case that n is prime.

7. (12 points) It is possible to find infinitely many positive integers n such that at least 40% (or 0.4) of the positive integers less than or equal to n are prime.

8. (12 points) If a is an integer such that $\gcd(a, 15) = 1$, then it must be the case that $a^{14} \equiv 1 \pmod{15}$.

9. (12 points) **PROOF QUESTION.** Note that $112 = 16 \cdot 7 = 2^4 7$.

Suppose that a, b, c are integers such that:

- 7 does not divide a;
- $ab \equiv 3 \pmod{16}$; and
- $ac \equiv 0 \pmod{112}$.

Prove that 112 divides c.