SA22
Math 112, Spring 2006

You may find the following definite integrals useful. (I.e., these are given.)

$$\int_0^{2\pi} \sin^2 x\,dx = \int_0^{2\pi} \cos^2 x\,dx = \pi,$$

$$\int_0^{\pi/2} \sin^2 x\,dx = \int_{\pi/2}^{\pi} \sin^2 x\,dx = \int_{\pi}^{3\pi/2} \sin^2 x\,dx = \int_{3\pi/2}^{2\pi} \sin^2 x\,dx = \frac{\pi}{4},$$

$$\int_0^{\pi/2} \cos^2 x\,dx = \int_{\pi/2}^{\pi} \cos^2 x\,dx = \int_{\pi}^{3\pi/2} \cos^2 x\,dx = \int_{3\pi/2}^{2\pi} \cos^2 x\,dx = \frac{\pi}{4}.$$

1. Let S be the surface $z = x^2 - y^2$, $-2 \le x \le 2$, $-1 \le y \le 3$, oriented by the **upwards** normal, and let

$$\mathbf{F}(x,y,z) = xy\mathbf{i} + z\mathbf{j} + y^3\mathbf{k} = (xy, z, y^3).$$

   Compute

$$\iint_S \mathbf{F} \cdot d\mathbf{S}.$$

2. Let S be the surface $z = 5 - x^2 - y^2$, $x^2 + y^2 \le 4$, oriented by the **upwards** normal, and let

$$\mathbf{F}(x,y,z) = 3x\mathbf{i} + x^2\mathbf{j} + z\mathbf{k} = (3x, x^2, z).$$

   Compute

$$\iint_S \mathbf{F} \cdot d\mathbf{S}.$$

   (Hint: Use polar coordinates.)

3. Verify Green's Theorem (vector form) for the case where $\mathbf{F}(x,y) = (x^2+y)\mathbf{i} - y^2\mathbf{j}$ and D is the unit disk $x^2 + y^2 \le 1$. (In other words, compute both sides of the equation in Green's Theorem and check that they are equal.)

4. (8.1) 6(a).

5. (8.1) 10.