

SA15
Math 112, Spring 2006

1. (6.1) 6.

2. Let $U = \{(r, \theta) \mid r \geq 0\}$, and let $f : U \rightarrow \mathbb{R}^2$ be the polar coordinates map:

$$f(r, \theta) = (r \cos \theta, r \sin \theta).$$

- (a) What is the image under f of the line $r = 4$ in the (r, θ) -plane? Is f one-to-one when restricted to this line? Draw the line in the (r, θ) -plane and its image in the (x, y) -plane.
- (b) Let D be the box in the (r, θ) -plane given by $0 \leq \theta \leq \frac{\pi}{2}$, $2 \leq r \leq 3$. Is f one-to-one when restricted to D ? Draw D in the (r, θ) -plane and its image in the (x, y) -plane.
- (c) Let E be the disc of radius 1, and center $r = 2$, $\theta = 0$. In other words, let

$$E = \{(r, \theta) \mid (r - 2)^2 + \theta^2 \leq 1\}.$$

Draw E in the (r, θ) -plane and its image in the (x, y) -plane.

3. Let $g : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ be the spherical coordinates map:

$$g(\rho, \theta, \varphi) = (\rho \cos \theta \sin \varphi, \rho \sin \theta \sin \varphi, \rho \cos \varphi).$$

Let E_1 be the box given by $1 \leq \rho \leq 2$, $0 \leq \theta \leq \frac{\pi}{6}$, $\frac{\pi}{6} \leq \varphi \leq \frac{\pi}{3}$, and let E_2 be the box given by $3 \leq \rho \leq 4$, $0 \leq \theta \leq \frac{\pi}{6}$, $\frac{\pi}{6} \leq \varphi \leq \frac{\pi}{3}$. Draw E_1 and E_2 in (ρ, θ, φ) -space and draw their images in (x, y, z) -space. Which image is bigger?