Topics for Exam 3
Math 112, Spring 2006

General information. Exam 3 will be a timed test of 50 minutes, covering 6.1–6.2, 4.3–4.4, 7.2–7.3, and 7.6 of the text. Most of the exam will be based on the homework assigned for those sections. If you can do all of that homework, and you know and understand all of the ideas behind it, you should be in good shape.

You are allowed to use a calculator and notes on **ONE** 3×5 note card (both sides).

As mentioned above, your first priority should be to understand the homework and quizzes and the ideas behind them. Besides the list of things you should know, below, you should also be familiar with everything specially emphasized in the text. If time permits, try to do some of the problems that have answers in the back of the book.

Section 6.1. Idea of a map; examples. One-to-one maps, onto maps; examples. Mappings that are both one-to-one and onto. Examples: polar coordinates, cylindrical coordinates, spherical coordinates. *Not in text, but related:* Pictures of 3-D examples.

Section 6.2. Idea of change of variables (map region forward, pull integral back). Jacobian determinant. Change of variables formula, double integrals; change of variables formula, triple integrals. Specific examples and fudge factors: polar coordinates, cylindrical coordinates, spherical coordinates.

Section 4.3. What a vector field is. How to draw a vector field. Specific examples: radial outward vector fields, swirlies of various types. Gradient vector fields (definition). Flow lines.

Section 4.4. Definitions: $\operatorname{curl}\mathbf{F}$, $\operatorname{div}\mathbf{F}$. "Del" notation: ∇f, $\nabla \times \mathbf{F}$, $\nabla \cdot \mathbf{F}$. Interpretation: $\operatorname{curl}\mathbf{F}$ gives swirliness, $\operatorname{div}\mathbf{F}$ measures outflow from little box. Curl of a gradient is zero; div of a curl is zero. Curl test for gradient vector fields; div test for curl vector fields.

Section 7.2. Idea of the line integral: work as motivation for definition. Definition of line integral; direct calculations of line integrals. Examples. Line integral only depends on curve and direction of travel, not parameterization (Theorem 1). Line integrals over geometric curves; simple closed curves; algebra of oriented curves. Line integral of a gradient vector field.

Section 7.3. Parameterized surface: definition, examples. Tangent vectors to parameterized surfaces. Normal vectors to parameterized surfaces; tangent planes to parameterized surfaces.

Section 7.6. Idea of surface integral as flow through a surface. Definition of surface integral over a parameterized surface (map region forward, pull integral back). Orientation of a surface; case of a graph $z = f(x, y)$. Surface integral only depends on surface and its orientation (Theorem 4). Examples from physics: flow, Gauss' law. Fudge factor for special case of a graph. Summary on pp. 496–497 (surface integrals of vector fields only).

Not on exam. (6.1) Linear maps (as particular examples). (4.3) Conservation of energy and escaping Earth's gravitational field. (4.4) Vector identities. (7.2) Reparameterization of path integrals; the $d\mathbf{r}$ notation for line integrals. (7.3) Technical details of regular surfaces. (7.6) Relation between surface integrals and scalar integrals.