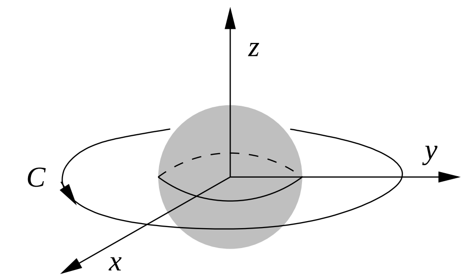
Paragraph HW 09
Calculating line integrals using Stokes' Theorem
Math 112, Spring 2006

1. Let $\mathbf{F}$ be a vector field such that:

   - The domain of $\mathbf{F}$ is $\{(x, y, z) \mid x^2 + y^2 + z^2 > 2\}$, i.e., $\mathbb{R}^3$ minus the closed ball of radius 2 centered at the origin.
   - $\operatorname{curl} \mathbf{F} = \mathbf{0}$ for all points in the domain of $\mathbf{F}$.



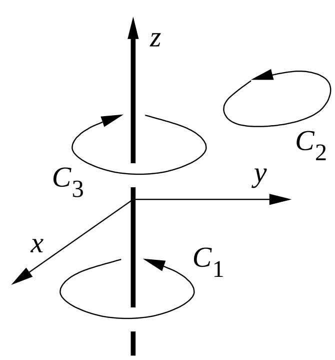
   Let C be the circle of radius 5 and center $(0, 0, 0)$ in the xy-plane in $\mathbb{R}^3$, oriented counterclockwise, as shown above. (The shaded area in the picture represents the points in $\mathbb{R}^3$ where $\mathbf{F}$ is not defined.) Use Stokes' Theorem to explain, using words and pictures, how we can be sure that

$$\int_C \mathbf{F} \cdot d\mathbf{s} = 0.$$

2. Let $\mathbf{F}$ be a vector field such that:

   - The domain of $\mathbf{F}$ is $\mathbb{R}^3$ minus the z-axis.
   - $\operatorname{curl} \mathbf{F} = \mathbf{0}$ for all points in the domain of $\mathbf{F}$.

   Also, let C_1, C_2, and C_3 be the curves shown below. Note that the z-axis, where $\mathbf{F}$ is not defined, is indicated by a bold line.



   Finally, suppose that

$$\int_{C_1} \mathbf{F} \cdot d\mathbf{s} = 13.$$

(continued)

(a) Explain why the fact that $\int_{C_1} \mathbf{F} \cdot d\mathbf{s} = 13$ does not contradict Stokes' Theorem.

(b) Find the value of $\int_{C_2} \mathbf{F} \cdot d\mathbf{s}$. Briefly **explain** your answer, using words and pictures.

(c) Find the value of $\int_{C_3} \mathbf{F} \cdot d\mathbf{s}$. Briefly **explain** your answer, using words and pictures. (Suggestion: Look for an oriented surface whose boundary is equal to $C_1 + C_3$. What does Stokes' Theorem say in this case?)