Paragraph HW 08
Divergence and surface integrals
Math 112, Spring 2006

Consider the following vector fields:

$$\mathbf{F}_1 = z\mathbf{k},$$
$$\mathbf{F}_2 = x\mathbf{j} + y\mathbf{k},$$
$$\mathbf{F}_3 = \frac{x}{(x^2+y^2+z^2)^{3/2}}\mathbf{i} + \frac{y}{(x^2+y^2+z^2)^{3/2}}\mathbf{j} + \frac{z}{(x^2+y^2+z^2)^{3/2}}\mathbf{k}.$$

Let S_1 be the unit sphere centered at the origin, oriented by the outward normal, and let S_2 be the sphere of radius 5 centered at the origin, oriented by the outward normal.

1. For each $\mathbf{F}_i$, $i = 1, 2, 3$, do each of the following:
   (a) Calculate $\operatorname{div} \mathbf{F}_i$.
   (b) Calculate $\iint_{S_1} \mathbf{F}_i \cdot d\mathbf{S}$.
   (c) Calculate $\iint_{S_2} \mathbf{F}_i \cdot d\mathbf{S}$.

2. Suppose we have a vector field $\mathbf{F}$ such that, at every point in $\mathbb{R}^3$ except possibly $(0, 0, 0)$, $\operatorname{div} \mathbf{F} = 0$.
   (a) Does it always seem to be the case that $\iint_{S_1} \mathbf{F} \cdot d\mathbf{S} = 0$? Explain your answer, using appropriate examples from the previous problem.
   (b) Does it always seem to be the case that $\iint_{S_1} \mathbf{F} \cdot d\mathbf{S} = \iint_{S_2} \mathbf{F} \cdot d\mathbf{S}$? Explain your answer, using appropriate examples from the previous problem.